NAC 2015

Early Numeracy & Beginning Math Concepts

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Pennsylvania Training and Technical Assistance Network

PaTTAN's Mission

The mission of the Pennsylvania Training and Technical Assistance Network (PaTTAN) is to support the efforts and initiatives of the Bureau of Special Education, and to build the capacity of local educational agencies to serve students who receive special education services.

2

PDE's Commitment to Least Restrictive Environment (LRE)

Our goal for each child is to ensure Individualized Education Program (IEP) teams begin with the general education setting with the use of Supplementary Aids and Services before considering a more restrictive environment.

3

Session Outline

Early numeracy **concepts and skills are essential** for continued achievement in mathematics. Structuring students' earliest experiences with mathematics in a CRA sequence can help them conceptualize the **concept of number** and provide for more **fluent and flexible counting and computation.**

Objectives

- Participants will be able to model whole numbers using **place value** concepts.
- Participants will understand the importance of the ability to **subitize** and apply to skill to teach addition and subtraction.
- Participants will be able to **utilize various tools** (ten-frame, rek-n-rek, etc.) to model mathematical concepts.

Tech Connection

wiggio.com

group name: **pattan math**

password: **ptnmath**

5

Session Outline

1. Quasi-History of Math
2. **Concept of Number**
3. **Number Bonds**
4. **Ten-Frames**
5. **Rekenrek**
6. **Fractions**

Early Numeracy…

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PA Core: *Early Numbers/Operations Standards*

	Grade PreK 2.1.PreK	Grade K 2.1.K	Grade 1 2.1.1
Pennsylvania's public schools shall teach, challenge, and support every student to real			
(A) Counting & Cardinality	CC.2.1.PreK.A.1 Know number names and the count sequence.	CC.2.1.K.A.1 Know number names and write and recite the count sequence.	*Intentionally Blank*
	CC.2.1.PreK.A.2 Count to tell the number of objects.	CC.2.1.K.A.2 Apply one-to-one correspondence to count the number of objects.	
	CC.2.1.PreK.A.3 Compare numbers.	CC.2.1.K.A.3 Apply the concept of magnitude to compare numbers and quantities.	
(B) Numbers & Operations in Base Ten	*Intentionally Blank*	CC.2.1.K.B.1 Use place value to compose and decompose numbers within 19.	CC.2.1.1.B.1 Extend the counting sequence to read and write numerals to represent objects.
		Intentionally Blank	CC.2.1.1.B.2 Use place-value concepts to represent amounts of tens and ones and to compare two digit numbers.
			CC.2.1.1.B.3 Use place-value concepts and properties of operations to add and subtract within 100.

	Grade PreK 2.2.PreK	Grade K 2.2.K	Grade 1 2.2.1
Pennsylvania's public schools shall teach, challenge, and support every studen			
(A) Operations and Algebraic Thinking	CC.2.2.PreK.A.1 Understand addition as putting together and adding to, and understand subtraction as taking apart and taking from.	CC.2.2.K.A.1 Extend the concepts of putting together and taking apart to add and subtract within 10.	CC.2.2.1.A.1 Represent and solve problems involving addition and subtraction within 20.
	Intentionally Blank	*Intentionally Blank*	CC.2.2.1.A.2 Understand and apply properties of operations and the relationship between addition and subtraction.
	Intentionally Blank	*Intentionally Blank*	*Intentionally Blank*
	Intentionally Blank	*Intentionally Blank*	*Intentionally Blank*

A Quasi-History

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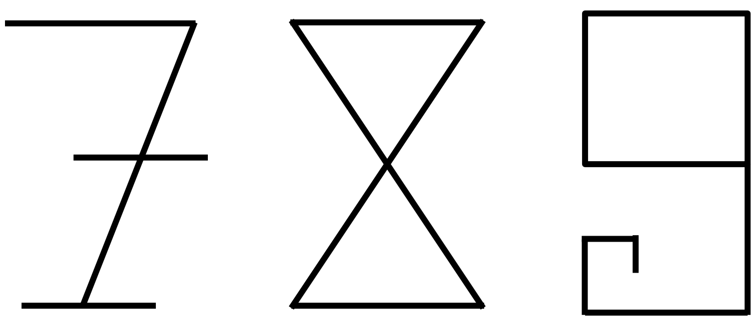
What is this?

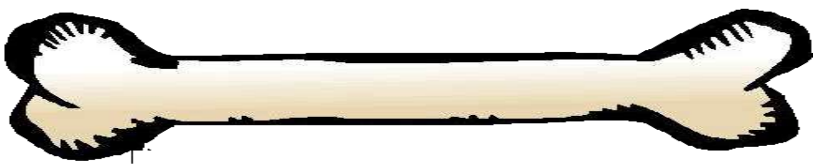


not factual…

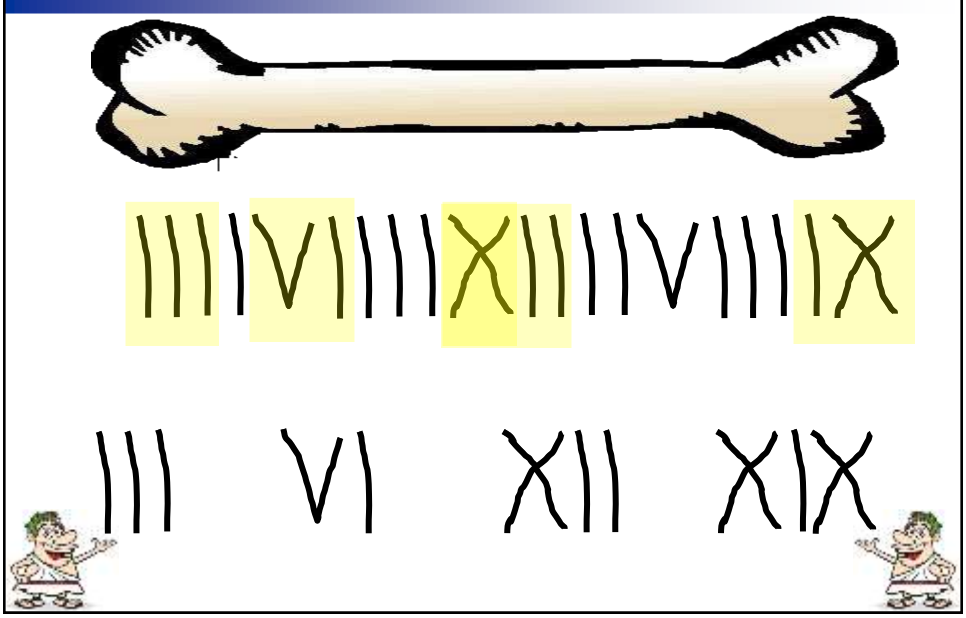
A Quasi-History of Number

20,000 years ago…
- Tally Systems
- Grouping structure

(Czechoslovakia, 1937)



A Quasi-History of Number



A Quasi-History of Number

Tally Systems

Grouping structure

(Czechoslovakia)

Uruk c. 4000BC

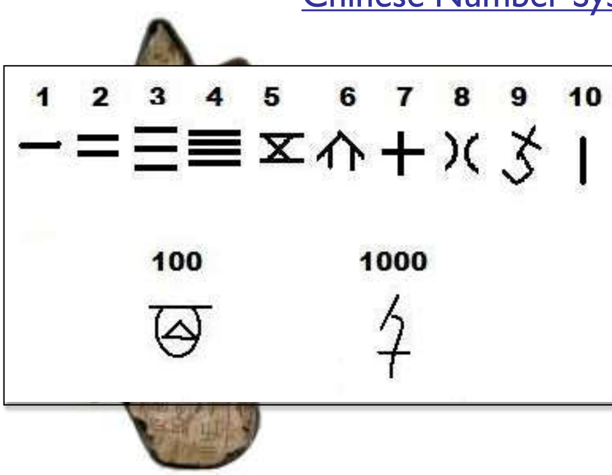
Place tokens in ball
Bake to prevent tampering
Mark outside with symbols to preserve records

Some time passes… local systems converge

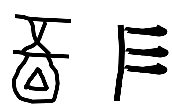
Babylonian Number Systems c.1950 BC

A Quasi-History of Number

Chinese Number System



BASE-10



$5 \cdot 10^2$ $1 \cdot 10^1$ $3 \cdot 10^0$

$500 + 10 + 3$

513

(oracle bone script c. 1400 BC)

Positional Base System

A Quasi-History of Number

Germanic / Irish / Britain / Roman (Base 12)

$$12\ troy\ oz. = 1\ troy\ lb.$$
$$12\ pence = 1\ shilling$$
$$Dozen = 12$$
$$Gross = 12 \times 12 = 144$$
$$Great\ Gross = 12 \times 12 \times 12 = 1728$$

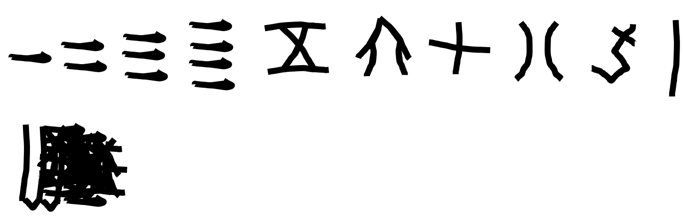
<u>TIME</u>

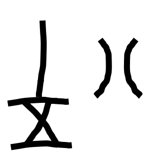
$12 \times 2\ hours = 1\ day$ $\qquad$ $12\ months = 1\ year$

$12\ zodiac\ signs$ $\qquad$ $Chinese\ Calendar$

$\boldsymbol{Babylon} \ldots\ \mathbf{60 \div 5 = 12}$!

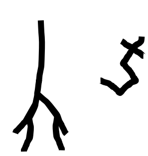
"Shang-style" Counting





$50 + 8$

$5\ tens, 8$



$60 + 9$

$6\ tens, 9$

Language & Number


Numerical Mechanisms and Children's Concept of Numbers

The numeric systems invented vary across time and place, and there is no doubt that the properties of such a system can facilitate or impede the development of children's mathematical understanding.

Chinese (and Asian languages based on ancient Chinese) are organized such that the numerical names are compatible with the traditional 10-base numeration system. So spoken numbers correspond exactly to their written equivalent: 15 is spoken as "ten five" and 57 as "five ten seven."

Most European systems of number words are irregular up to 100. For example in French, 92 is said as "four twenty twelve," corresponding to $4 \times 20 + 12$.

The more complicated the number word system is, the harder it is for children to learn the counting sequence.

http://web.media.mit.edu/~stefanm/society/som_final.html

Interpreting Numbers

1. What is this number?

2. What is the meaning of this number?

3264

18

Decimal *(base 10)*

$$3264$$

10^3 10^2 10^1 10^0

thousands hundreds tens ones

$$(3 \times 10^3) + (2 \times 10^2) + (6 \times 10^1) + (4 \times 10^0)$$

$$(3 \times 1000) + (2 \times 100) + (6 \times 10) + (4 \times 1)$$

$$3000 + 200 + 60 + 4$$

!

19

Language of Number

A B C D E F G H I J K L M
N O P Q R S T U V W X Y Z

26 symbols — *name* $\Longrightarrow$ *sound* $\Longrightarrow$ *word*

Carla piece chocolate

0 1 2 3 4 5 6 7 8 9

10 symbols — *name* $\Longrightarrow$ *quantity* $\Longrightarrow$ *number*

207 71 -7 0.7 1/7

Symbols & Meaning

- Two ways to understand letters…
  - "B" is the letter "bee" and makes the sound */b/*
- What about numbers?
  - Names are taught
  - Meaning is based on place value (base 10)

Number Names & Meanings

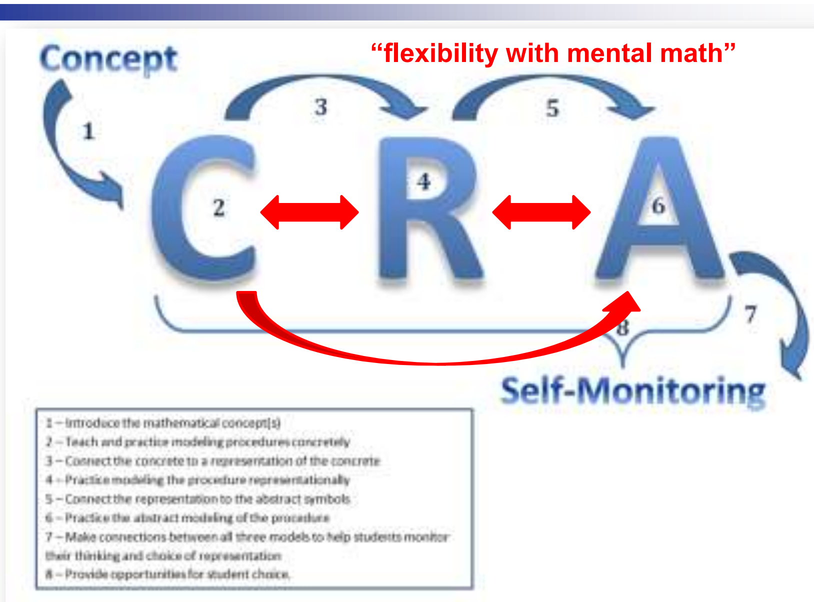
#	Name	Meaning	#	Name	Meaning
0	Zero	None	20	Twenty	Two tens
1	One	One	21	Twenty-one	Two tens, one
2	Two	Two	22	Twenty-two	Two tens, two
3	Three	Three	23	Twenty-three	Two tens, three
4	Four	Four	24	Twenty-four	Two tens, four
5	Five	Five	25	Twenty-five	Two tens, five
6	Six	Six	26	Twenty-six	Two tens, six
7	Seven	Seven	27	Twenty-seven	Two tens, seven
8	Eight	Eight	28	Twenty-eight	Two tens, eight
9	Nine	Nine	29	Twenty-nine	Two tens, nine
10	Ten	One ten	30	Thirty	Three tens
11	Eleven	One ten, One	31	Thirty-one	Three tens, one
12	Twelve	One ten, Two	32	Thirty-two	Three tens, two
13	Thirteen	One ten, Three	Other examples		
14	Fourteen	One ten, Four	48	Forty-eight	Four tens, eight
15	Fifteen	One ten, Five	53	Fifty-three	Five tens, three
16	Sixteen	One ten, Six	62	Sixty-two	Six tens, two
17	Seventeen	One ten, Seven	75	Seventy-five	Seven tens, five
18	Eighteen	One ten, Eight	81	Eighty-one	Eights tens, one
19	Nineteen	One ten, Nine	99	Ninety-nine	Nine tens, nine

What is Number Sense?

"a child's fluidity and flexibility with numbers, the sense of what numbers mean, and an ability to perform mental mathematics and to look at the world and make comparisons"

(Gersten & Chard, 1999)

CRA Sequence of Instruction



CRA

- Concrete (sense making by moving)
- Representational (sense making by drawing)
- Abstract (sense making with symbols)

CONSISTENT LANGUAGE

Rationale – Doing What Works

Research-based studies show that students who use concrete materials develop **more precise and more comprehensive mental representations**, often show more motivation and on-task behavior, understand mathematical ideas, and better apply these ideas to life situations.

(Harrison, & Harrison, 1986)
(Suydam & Higgins, 1977)

Concrete-Representational-Abstract Instructional Approach Summary Report–
The Access Center, American Institutes for Research, Washington, DC

http://dww.ed.gov DOINGWHATWORKS

Why would CRA be effective?

- Multimodal forms of math acquisition to aid memory and retrieval
- Meaningful manipulations of materials allows students to rationalize abstract mathematics
- Procedural accuracy; provides alternate to algorithm memorization

(Witzel, Riccomini, & Scheider, 2008)

Other Research.

- Direct Instruction
- Errorless Teaching
- Formative Assessment
- Correct Feedback
- Improved Teacher Content Knowledge
  - Task Analyze
  - Instruct on Specific Skills or Process
  - Monitor progress
  - Correct errors

Something here…

- Multimodal forms of math acquisition to aid memory and retrieval

- Meaningful manipulations of materials allows students to rationalize abstract mathematics

- Procedural accuracy; provides alternate to algorithm memorization

Students having difficulties with math…

- Counting seen as rote, mechanical, left to right, 1: 1 correspondence only; INEFFECTIVE

- Automaticity problems take up working memory, inhibit discourse & algebraic thinking

(Gersten, Jordan, & Flojo, 2005)

Concept of Number

"What does three really mean?
What is three-ness"

-M

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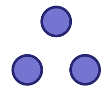
What is Number Sense?

"a child's fluidity and flexibility with numbers, the sense of what numbers mean, and an ability to perform mental mathematics and to look at the world and make comparisons"

(Gersten & Chard, 1999)

What does "3" really mean?

3 three *"three"*  

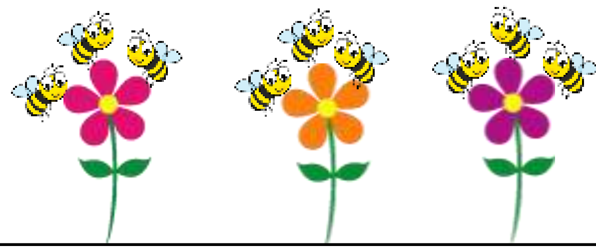
"1 ... 2 ... 3! "

"one more than 2" *"one less than 4"*

"is between… " *"is more than… "*

"is less than… "

"is the same as… "

3 units 

Teaching each symbol or Teaching the collection

Each Symbol

- Name – Meaning – Quantity
- Ability to Subitize

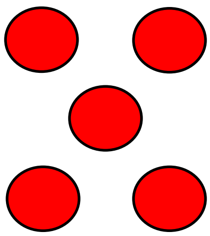
Collection

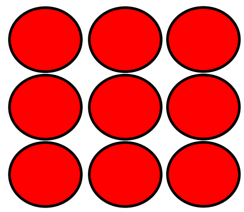
- Count Sequence
- Magnitude
- Missing Number
- Applications

Subitize

The ability to see a quantity and know how many, without "counting."

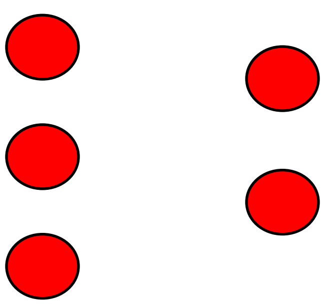
Perceptual and **Conceptual**





Subitizing & Conceptual Counting

$$3 + 2 = \mathbf{5}$$



Elementary Classroom – Conceptual Addition

VIDEO

Teaching each symbol or Teaching the collection

Each Symbol

- Name – Meaning – Quantity
- Ability to Subitize

Collection

- Count Sequence
- Magnitude
- Missing Number
- Applications

Basic Principles of Counting

Numerical Mechanisms and Children's Concept of Numbers

One-to-one – Counting one "thing" at a time; transfer from uncounted group to counted group ($\mathbf{1:1}\ \boldsymbol{Correspondance}$)

Stable-order – Establishes consistent sequence

Cardinal – The last count represent the quantity in the counted group ($\boldsymbol{Cardinality}$)

Abstraction – applying counting to like objects, actions, sounds, etc…

Order-irrelevance – Can count in any order

Stages of Early Arithmetical Learning

The **Stages of Early Arithmetical Learning** (SEAL) classifies the various strategies used by children into six stages:

Stage	Indicators
Stage 0: Emergent Counting	Cannot count visible items The child may not know the number words. The child cannot coordinate number words with items.
Stage 1: Perceptual Counting	Can count perceived items May involve seeing, hearing or feeling items.
Stage 2: Figurative Counting	Can count the total of two collections. Counts from one.
Stage 3: Initial Number Sequence	Child uses and understands counting-on rather than counting-from-one. Uses counting on to solve addition and missing addend tasks. May use count-down-from strategies.
Stage 4: Intermediate Number Sequence	The child uses and understands: • count-down-from strategies • count-down-to strategies The child can choose the most efficient strategy.
Stage 5:Facile Number Sequence	The child uses a range of non-count by one strategies: • Compensation • Using known results • Adding to ten • Commutativity • Subtraction as the inverse of addition • Awareness of ten as a teen number

Wright, R., Martland. J, Stafford, A., & Stanger, G. (2006). *Teaching Number: Advancing Children's Skills and Strategies.* London: Sage.

What is Number Sense?

"a child's fluidity and flexibility with numbers, the sense of what numbers mean, and an ability to perform mental mathematics and to look at the world and make comparisons"

(Gersten & Chard, 1999)

From Counting to Computation

... or more efficient counting

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What is the sum?

Strategies

Compensation

$$1000 + 32 = 1032 - 1$$

$$999 + 32 = 1031$$

$$999 + 1 + 31 = 1000 + 31$$

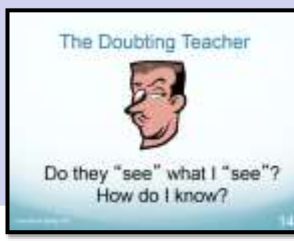
Decomposition

Decomposition & Compensation

Strategies

Decomposition – decomposing numbers to compute faster

- ✓ **make a 5**
- ✓ **make a 10**
- ✓ **doubles (±1)**



Compensation – Adjust the problem to compute, then readjust the answer

- ✓ **may utilize known facts.**

The Mathematics Framework, Appendix F

Levels	8 + 6 = 14	14 - 8 = 6
Level 1: Count all	Count All	Take Away
Level 2: Count on	Count On	Think +
Level 3: Recompose Make a ten (general): one addend breaks apart to make 10 with the other addend Make a ten (from 5's within each addend)	Make 5/10	From 5/10
Doubles ± n	6 + 8 = 6 + 6 + 2 = 12 + 2 = 14	

Note: Many children attempt to count down for subtraction, but counting down is difficult and error-prone. Children are much more successful with counting on; it makes subtraction as easy as addition.

Number Bonds

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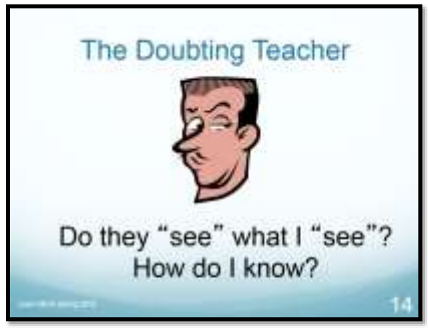
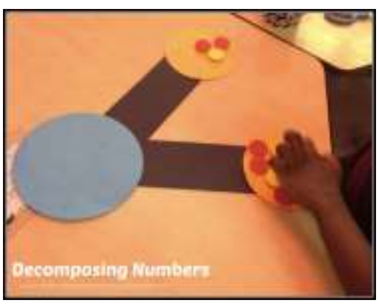
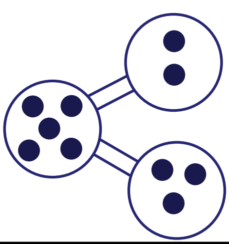
Composing & Decomposing Numbers

C



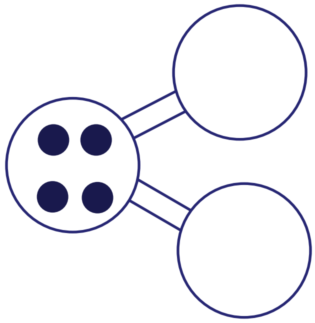
Composing & Decomposing Numbers





C R A

Number Bonds – Fact Families

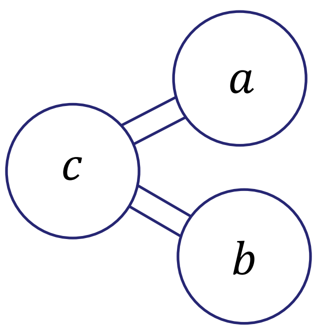


$$4 = 2 + 2$$

$$4 = 1 + 3$$

$$4 = 0 + 4$$

Number Bonds – Fact Families



$$a + b = c$$

$$a + b = ?$$

$$\left.\begin{aligned} a + ? &= c \\ ? + b &= c \end{aligned}\right\} c - a = ?$$

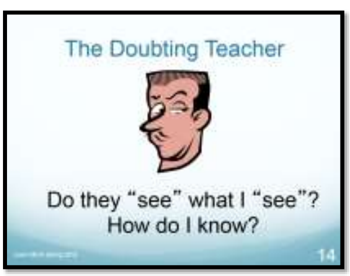
$$c - ? = b$$

Concrete/Representational

Modeling

Partner Practice (C or R)

- Count on
- Making 5
- Making 10
- Doubles (± 1)



$2 + 3$

$3 + 6$

$7 + 2$

$1 + 7$

$4 + 3$

$8 + 3$

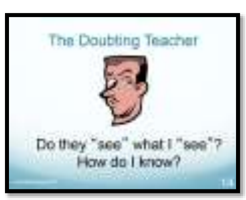
$2 + 9$

Concrete/Representational

Modeling

Partner Practice (C or R)

- Take Away
- Count on (Think $+$)
  - Missing addend
- Compensation
  - $From\ 5$
  - $From\ 10$
- Doubles (± 1)



$3-1$

$4-2$

$6-4$

$8-7$

$4-2$

$8-4$

$9-3$

Ten-Frames

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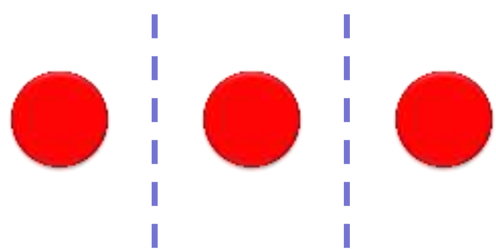
Decomposition



Decomposition

$$\mathbf{2 + 1 =}$$

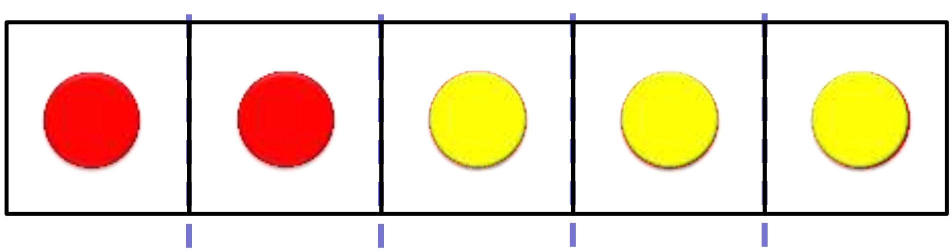
$$\begin{array}{r} \mathbf{2} \\ \underline{\mathbf{+1}} \end{array}$$

"two and one make ..."

"two plus one makes ..."

"two plus one equals ..."

Decomposition



see the parts & see the whole

Purpose of 10-frame

$\mathbf{5} - \boldsymbol{frame}$

$\mathbf{10} - \boldsymbol{frame}$

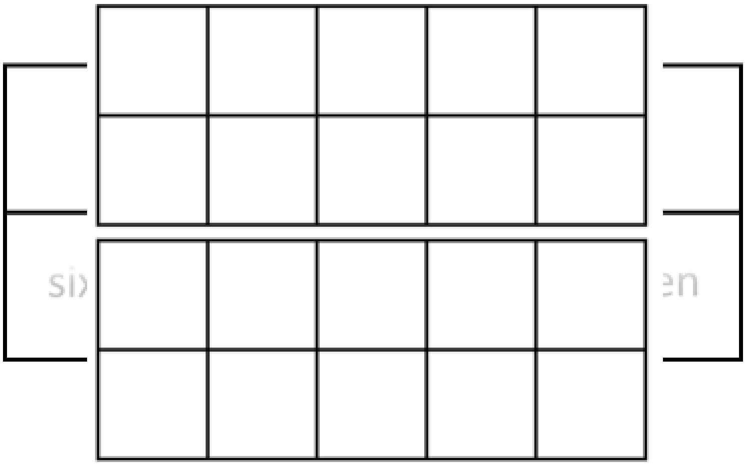
$\mathbf{two\ 10} - \boldsymbol{frames}$

- See sets of 5
- See sets of 10
- Organize in rectangular array
  - Subitize
- Reduces need to "count"
- Visually decompose numbers in sets of 5

Subitizing the 10-frame support

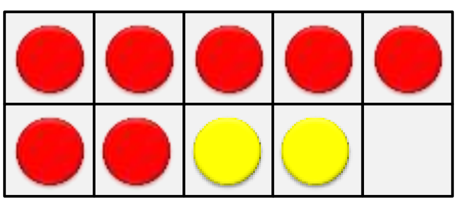
VIDEO

Ten-Frame Variations



Help students Subitize on the $10 - frame$.

What do you see?



$$\mathbf{9}$$

$$\mathbf{7+2}$$

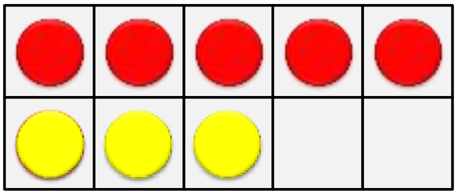
$$\mathbf{5+2+2}$$

$$\mathbf{9-2}$$

$$\mathbf{5+4}$$

$$\mathbf{10-1}$$

Modeling on a ten frame

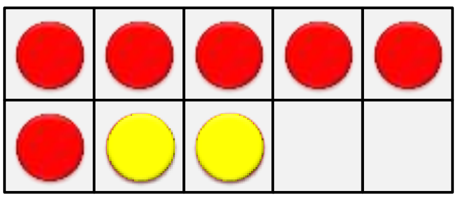


Count on

$6 + 2 = 8$

$5 + 3 = 8$

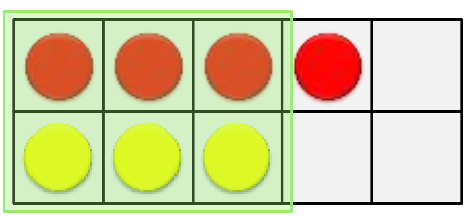
Make 5



$8 - 2 = 6$

Count on (Think +)

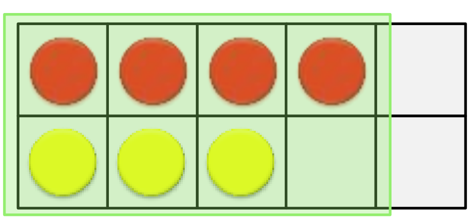
Modeling on a ten frame



$4 + 3 = 7$

$6 + 1 = 7$

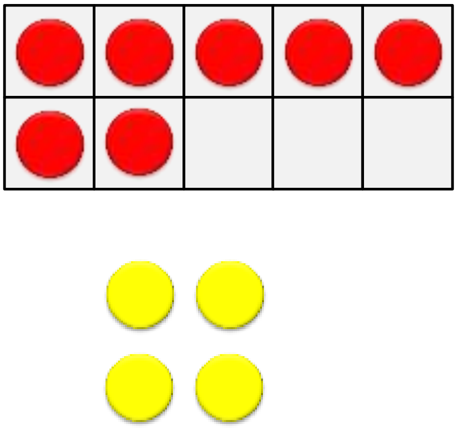
Doubles +1



$8 - 1 = 7$

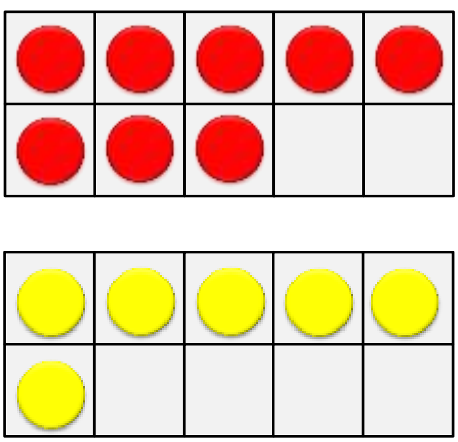
Doubles -1

Modeling on a ten frame — Making ten



$$7 + 4 =$$
$$3 + 1$$
$$10 + 1 =$$
$$11$$

Modeling on a ten frame — Making ten



$$8 + 6 =$$
$$2 + 4$$
$$10 + 4 =$$
$$14$$

Concrete Modeling

Partner Practice (C)

- Count All or Take Away
- Counting on
  - Subtraction: Missing addend
- Making 5
- Making 10
- Doubles (± 1)

$8 + 9$

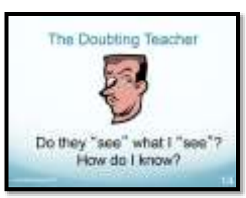
$4 + 6$

$7 - 3$

$3 + 4$

$7 + 8$

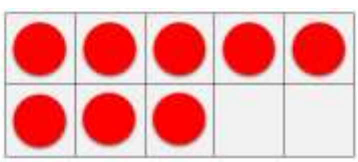
$12 - 4$



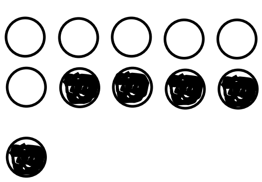
Teaching facts w/ 10-frame support

VIDEO

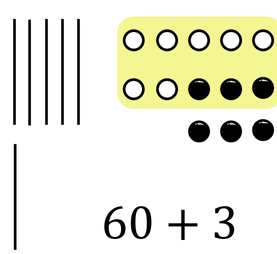
Ten-Frame Progression



$$\begin{array}{r} {\scriptstyle 1} \\ 27 \\ +\ 36 \\ \hline 63 \end{array}$$



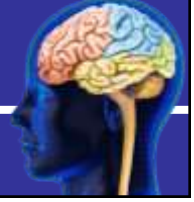
$$\begin{array}{r} 6 \\ +\,5 \\ \hline 10 \\ +\,1 \\ \hline 11 \end{array}$$



$60 + 3$

Ten Frame Ideas

Rekenrek



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Vocabulary

Rekenrek (wreck-n-wreck)

The Rekenrek (also called an arithmetic rack) has emerged as perhaps the most powerful of all models for young learners.

Developed by mathematics education researchers at the highly regarded Freudenthal Institute in the Netherlands.

Designed to reflect the natural intuitions and informal strategies that young children bring to the study of numbers, addition, and subtraction.

Provides a visual model that encourages young learners to build numbers by

- groups of five
- groups of ten
- doubling and halving strategies
- counting-on from known addition/subtraction

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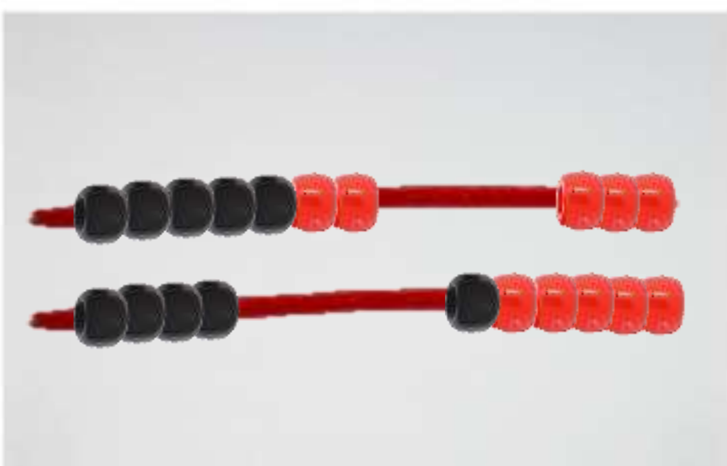
Rekenrek



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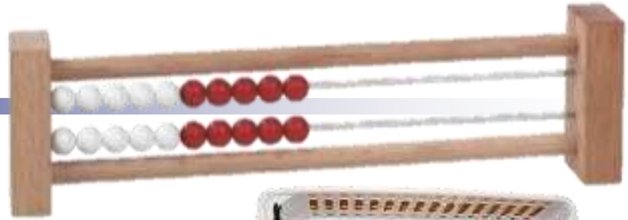
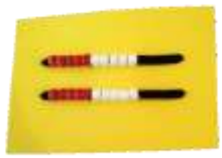
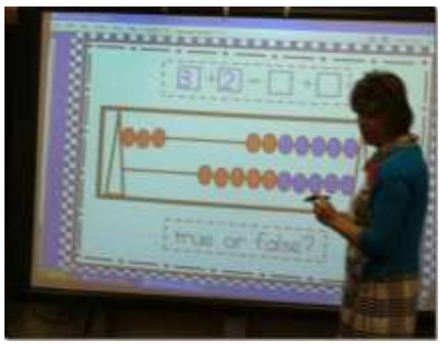
Some activities…

- **See & Slide** – Given #, make in 1 move.
- **Build a Number** – move first row, how many more on second row
- **Show Me** – Give number, make combination
- **Flash Attack** – Show beads, get number

Rekenrek

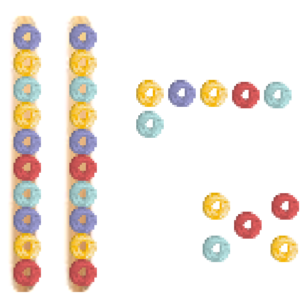






Number Rack for iPhone, iPad, and the Web

One more idea…



Fractions

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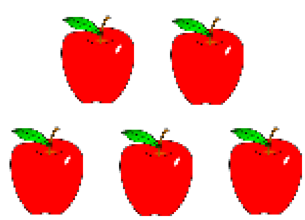
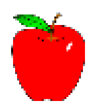
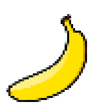
Early on in Fractions…



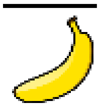
2 3 4 5



3 3 3

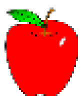
 3 3

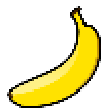
6



Early on in Fractions…

$$\frac{a}{b} = a \times \frac{1}{b} \qquad counting \text{ "} \frac{1}{b}\text{'s "}$$





$$\overline{}$$
$$3$$

$$\text{"}thirds\text{"}$$

Vocabulary

Fraction – from Latin: *fractus*, "broken"

$$\frac{numerator}{denominator}$$

enumerate
count

what is being counted
size / value
denomination

Interpreting Fractions – *"counting"*

VIDEO

Definitions / Models

Definitions

Part of whole

Ratio

Measurement

Operator/Quotient

Models

Area

circles, pattern blocks, graph/dot paper, paper folding

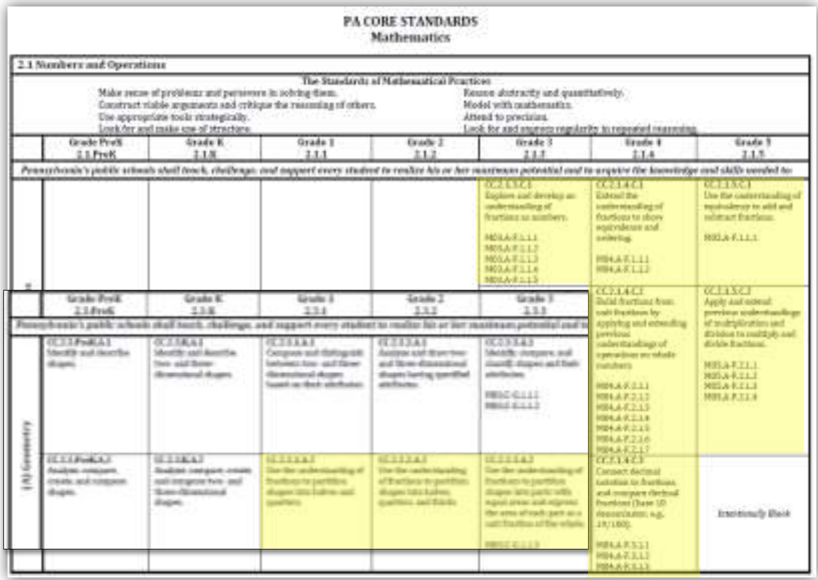
Length

Fraction strips, Cuisenaire rods, line segments, number line

Sets

Objects, groups or arrays

What & When?


PA CORE STANDARDS
Mathematics

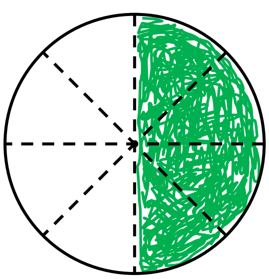
2.1 Numbers and Operations

The Standards of Mathematical Practices
Make sense of problems and persevere in solving them.
Construct viable arguments and critique the reasoning of others.
Use appropriate tools strategically.
Look for and make use of structure.
Reason abstractly and quantitatively.
Model with mathematics.
Attend to precision.
Look for and express regularity in repeated reasoning.

Early on in Fractions…

Geometry

$$\frac{a}{b} = a \times \frac{1}{b} \qquad counting\ "\ \frac{1}{b}'s\ "$$

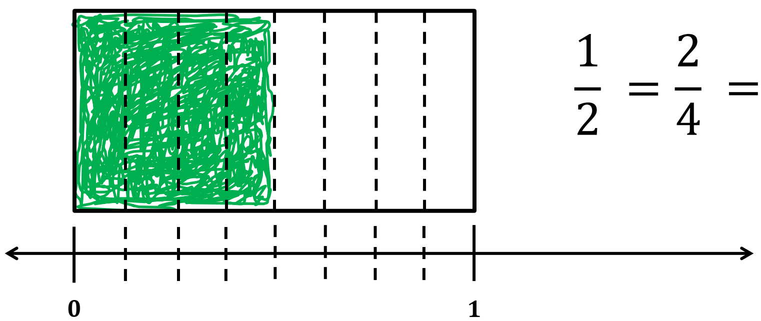


$$\frac{1}{2} = \frac{2}{4} = \frac{4}{8}$$

Early on in Fractions…

Geometry

$$\frac{a}{b} = a \times \frac{1}{b} \qquad counting\ "\ \frac{1}{b}'s\ "$$

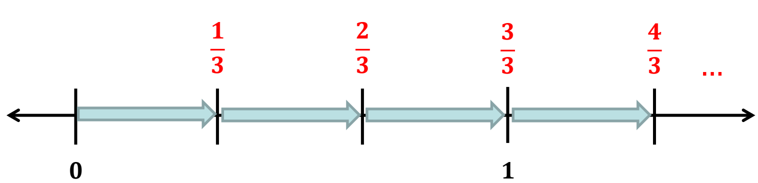


$$\frac{1}{2} = \frac{2}{4} = \frac{4}{8}$$

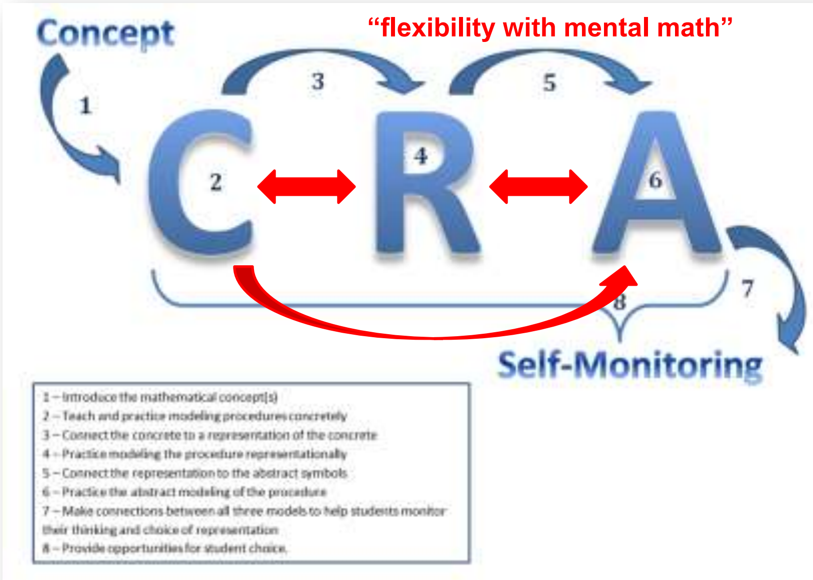
Early on in Fractions…

Number

$$\frac{a}{b} = a \times \frac{1}{b} \qquad counting\ "\ \frac{1}{b}'s\ "$$



CRA Sequence of Instruction



CRA

- Concrete (sense making by moving)
- Representational (sense making by drawing)
- Abstract (sense making with symbols)

CONSISTENT LANGUAGE

Resources

Pennsylvania Training and Technical Assistance Network

Tech Connection

wiggio.com

group name: **pattan math**

password: **ptnmath**

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CRA Days

	Pittsburgh	Harrisburg	King of Prussia
Early Numeracy	10/23/15	10/9/15	10/15/15
Addition & Subtraction	11/3/15	11/20/15	11/11/15
Multiplication & Division	2/25/16	2/16/16	2/25/16
Fractions	3/15/16	3/16/16	3/18/16
Integers & Equations	4/7/16	3/31/16	4/1/16

www.pattan.net

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DEPARTMENT OF EDUCATION
Bureau of Special Education
Pennsylvania Training and Technical Assistance Network

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PaTTAN

Commonwealth of Pennsylvania

Tom Wolf, Governor