NAC 2017

#ptnmath

Coherent Sequencing of Early Mathematics Content for Students with Autism

Jared Campbell
Willow Hozella
Educational Consultants, PaTTAN Harrisburg

Pennsylvania Training and Technical Assistance Network

Tech Connection

Sites.google.com/pattan.net/ptnmath

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PaTTAN's Mission

The mission of the Pennsylvania Training and Technical Assistance Network (PaTTAN) is to support the efforts and initiatives of the Bureau of Special Education, and to build the capacity of local educational agencies to serve students who receive special education services.

3

PDE's Commitment to Least Restrictive Environment (LRE)

Our goal for each child is to ensure Individualized Education Program (IEP) teams begin with the general education setting with the use of Supplementary Aids and Services before considering a more restrictive environment.

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Session Description

Foundational numeracy concepts are taken for granted in education. It is often assumed that students will possess certain skills before they even begin formal education in mathematics.

This assumption can create gaps in learning and lead to remediation, instead of altering the original instructional sequence to be more coherent.

Students with Autism often have delays in language acquisition, which leads to delayed instruction in mathematics. This delay in mathematics learning presents educators with a unique opportunity to redefine how we think about early numeracy concepts and design more coherent sequences in mathematics curricula.

Thinking differently about early numeracy?

- ~~Available curriculum/programs~~
- Identify skills
- Order skills logically
- Find associated prerequisites
- Teach to mastery/fluency/across exemplars/etc…

Session Outline

1. ABA Stuff
2. **Early Numeracy Sequencing**
3. **Counting Principles**
4. **Operations**

ABA Background

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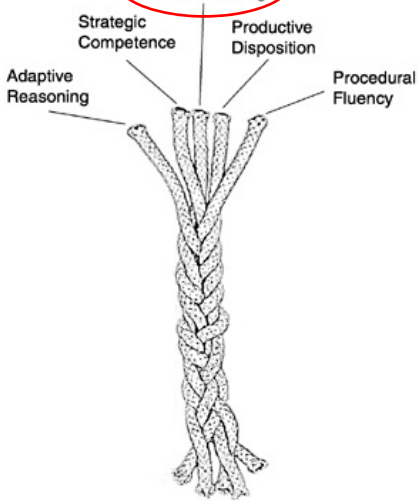
Pop Quiz!

Math is a

Language

5 Strands of Mathematical Proficiency

Prerequisites ← Conceptual Understanding → **Math Topic**



(NRC, 2001)

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What is **conceptual understanding?**

Extended Tacts

- Generalization must occur
  - Can apply to novel items without explicit teaching
  - Across…
    1. People
    2. Places
    3. Materials
    4. Instructions
    5. Time
- Feature/Function/Class
  - Tacting critical features may facilitate concept acquisition
- The tact is involved in the process of joint control which assists students in effective verbal recall and effective listener responding.

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What is **conceptual understanding?**

Atomic Repertoires

- New combination of skills applied to new behaviors
- Most of our spoken language is a result of ARs

What are the prerequisite skills needed for the atomic repertoires for the math content?

- Imitation
- Echoic
- Tacts
- Textual Behavior (reading texts/symbols)
- Transcriptive Behavior (copying text/symbols)
- Etc…

We must identify the skills in relation to content!

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"Concept Matrix"

Student (behavior)

Teacher (antecedent)

	Find digit	Write digit	Write text	Say number	Make pattern
Say number	LR	Trans.	Trans.	Echoic	LR
Show digit	MtS	Trans.	Trans.	IV	MtS
Show text	MtS	Trans.	Trans.	Text	
Show pattern	MtS	Trans.	Trans.	Tact	MtS

From this point on…

I am going to simplify the ABA Vocabulary so we can focus on the math.

You can still make connection/improvements if you have that level of background.

Early Numeracy

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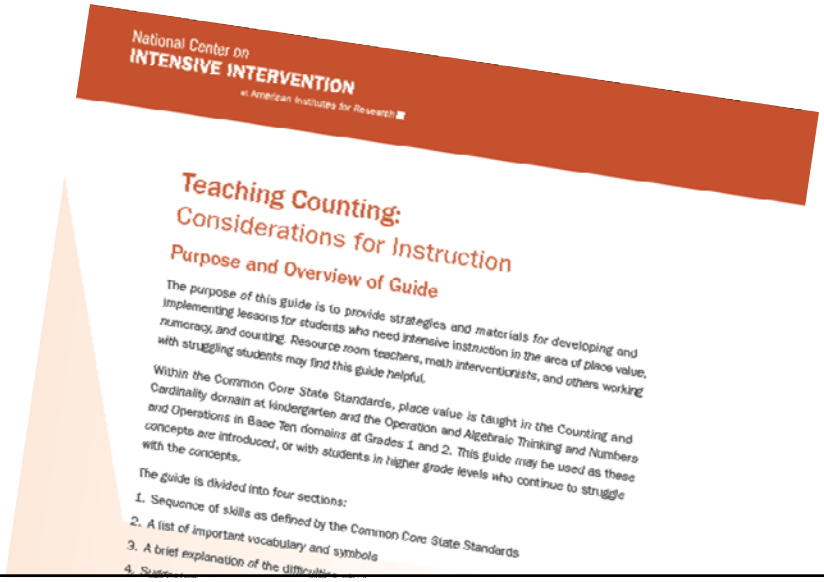
NCII: Teaching Counting

5 min.


National Center on
INTENSIVE INTERVENTION
at American Institutes for Research

Teaching Counting:
Considerations for Instruction

Purpose and Overview of Guide

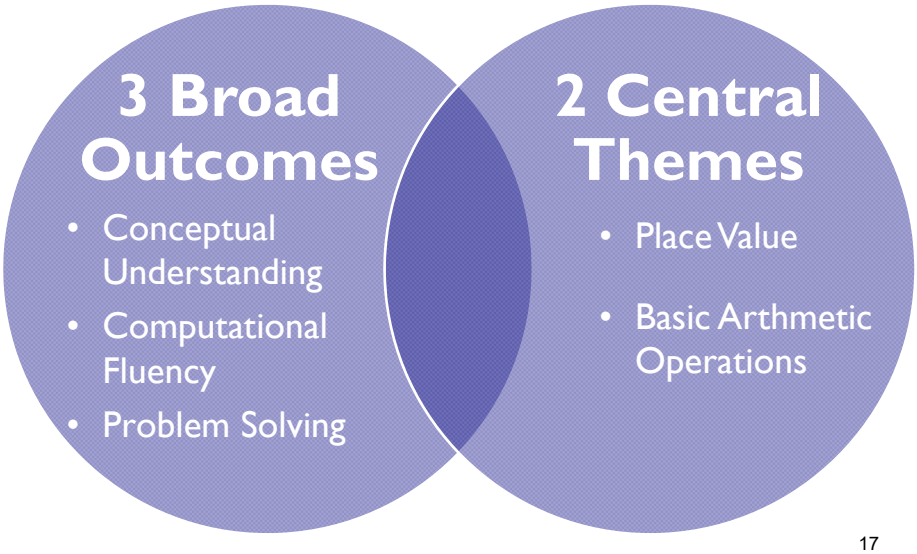
The purpose of this guide is to provide strategies and materials for developing and implementing lessons for students who need intensive instruction in the area of place value, numeracy, and counting. Resource room teachers, math interventionists, and others working with struggling students may find this guide helpful.

Within the Common Core State Standards, place value is taught in the Counting and Cardinality domain at kindergarten and the Operation and Algebraic Thinking and Numbers and Operations in Base Ten domains at Grades 1 and 2. This guide may be used as these concepts are introduced, or with students in higher grade levels who continue to struggle with the concepts.

The guide is divided into four sections:

1. Sequence of skills as defined by the Common Core State Standards
2. A list of important vocabulary and symbols
3. A brief explanation of the difficulties
4. Sug

Early Numeracy



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(Anderson, 2013)

Early Numeracy: Broad Outcomes

Conceptual Understanding (Willingham 2009)
- Undertanding meaning and rationale
- Logical, justifyable, knowing the *"why"*

Computational Fluency (NCTM 2000)
- Efficient, accurate methods to compute
- Accuracy, flexibility, understanding

Problem Solving (Schoenfeld 1992)
- Routine excersizes
- Reaching goal not immediately attainable, *"novel"*

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Early Numeracy: Central Themes

Place Value $(Base\ 10)$

- Single Digits
- Groups of ten
- Positional Base System

Basic Arithmetic Operations

- Addition/Subtraction
- Multiplication/Division

} **Inverse Operations**

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Basic Principles of Counting

Numerical Mechanisms and Children's Concept of Numbers

Natalie Marmasse, Aggelos Bletsas, Stefan Marti
The Media Laboratory
Massachusetts Institute of Technology
20 Ames Street, Cambridge, MA USA 02139
{nmarmasse | aggelos | stefanm}@media.mit.edu

3 **One-to-one** – Counting one "thing" at a time; transfer from uncounted group to counted group (**1: 1** ***Correspondance***)

1 **Cardinal** – The last count represent the quantity in the counted group (***Cardinality***)

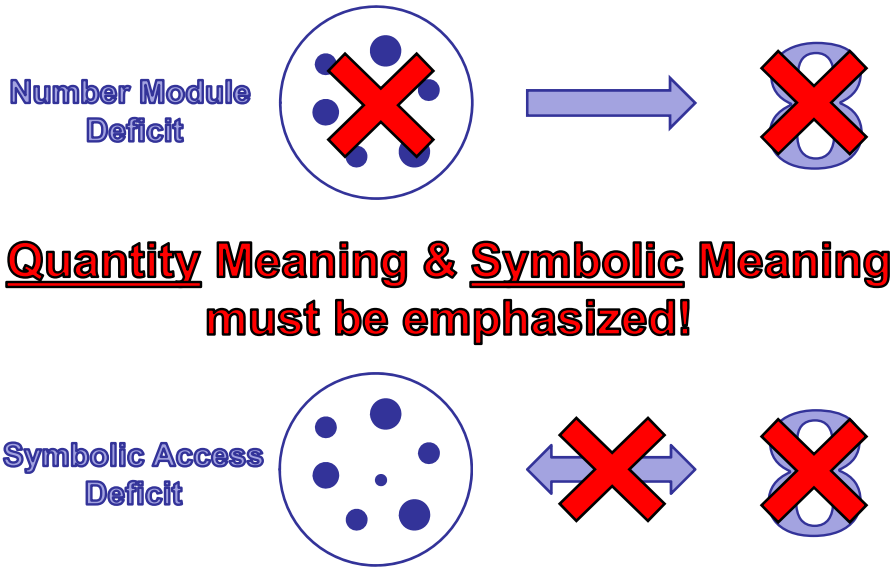
2 **Stable-order** – Establishes consistent sequence

Abstraction – applying counting to like objects, actions, sounds, etc…

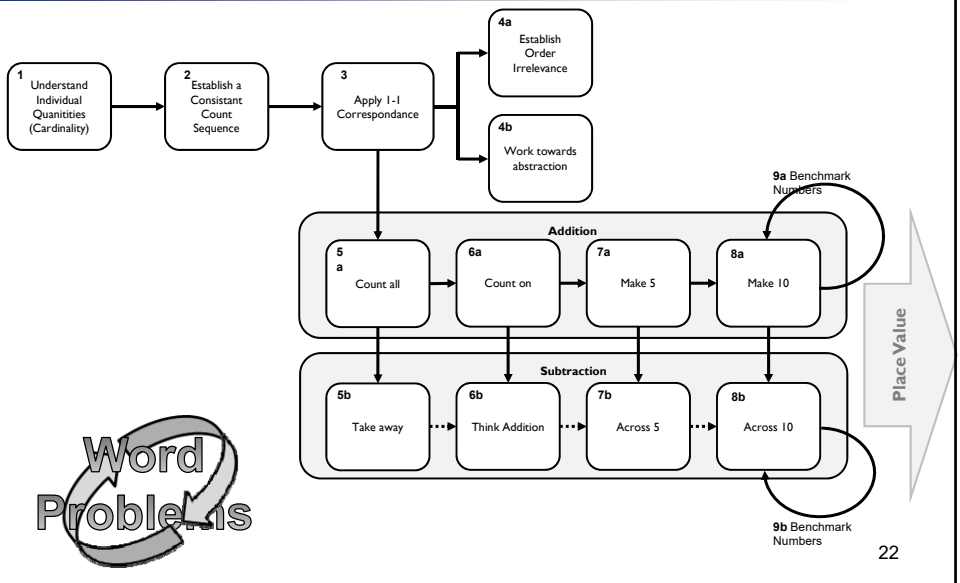
4/5

Order-irrelevance – Can count in any order

Developmental Dyscalculia



From Quantity to Computation



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Cardinality

"what numbers represent"

"What does three really mean?
What is three-ness"

-MM

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What does "3" really mean?

3 three *"three"*

"1 ... 2 ... 3!"

"one more than 2"

"one less than 4"

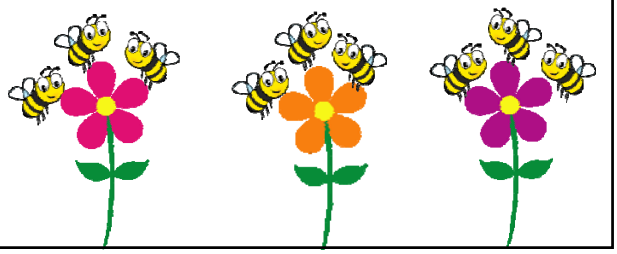
"is between… "

"is more than… "

"is less than… "

"is the same as… "

3 units



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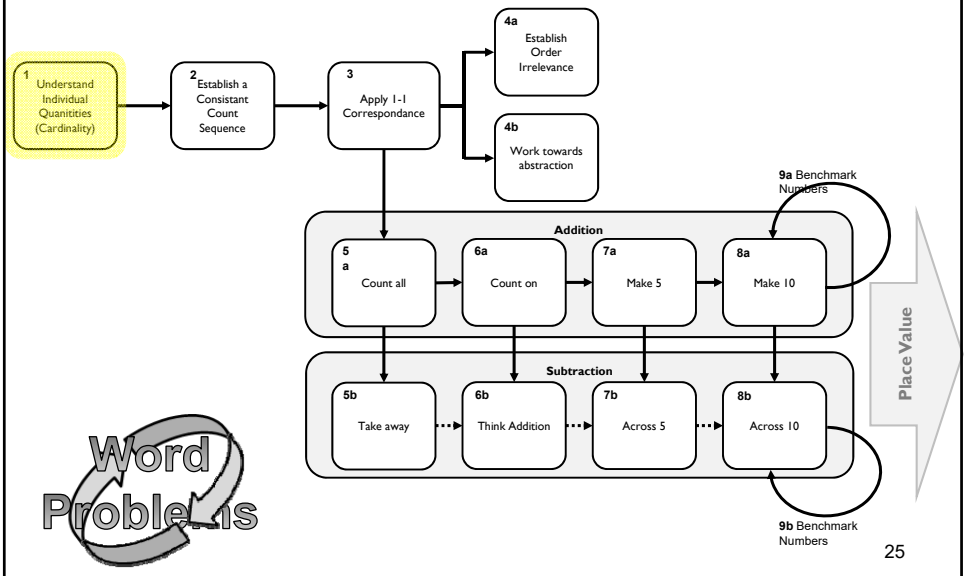

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Cardinality: the size of a set

- The number of elements in a set.

"A set of numbers, called S, contains the numbers 1, 3, 5, 7, and 9. The cardinality of the set S is 5."

if $S = \{1, 3, 5, 7, 9\}$ then $|S| = 5$

Cardinality begins by learning quantities/patterns.

Cardinality is enhanced with 1:1 Correspondence.

- Through 1:1 "counting"
- Through 1:1 "matching"

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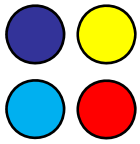
Cardinality: the size of a set

- The number of elements in a set.

"A set of dots, called D, contains the dots ●, ●, ●, and ●. The cardinality of the set D is 4."

if $D = \{$ ●, ●, ●, ● $\}$ **then** $|D| = 4$



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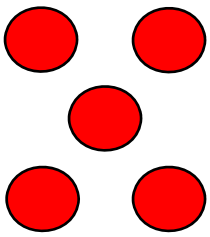
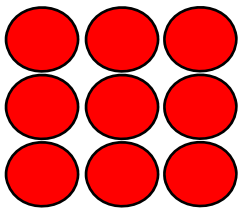
Subitization

The ability to see a quantity and know how many, without "counting."

Perceptual and **Conceptual**





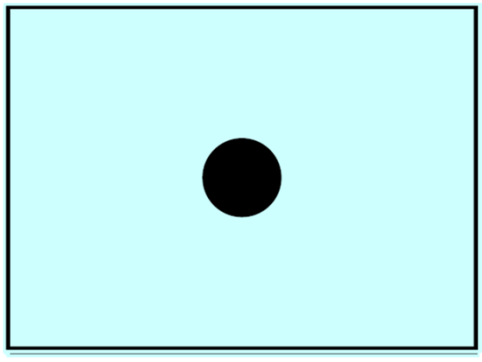
Subitization

Research indicated that dice patterns and rectangular arrays are the easiest for students to learn.

Don't go crazy!

Clements, D. H. (1999). Subitizing: What is it? Why teach it?. Teaching children mathematics, 5(7), 400.

Subitizing – "How Many?"



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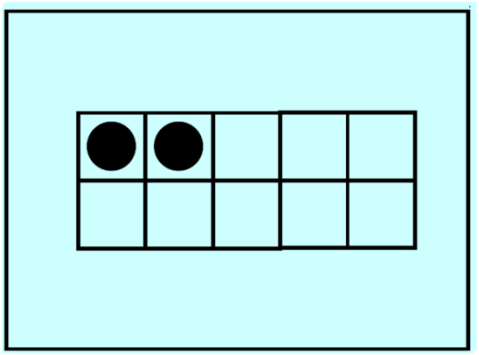
Subitizing – "How Many?"



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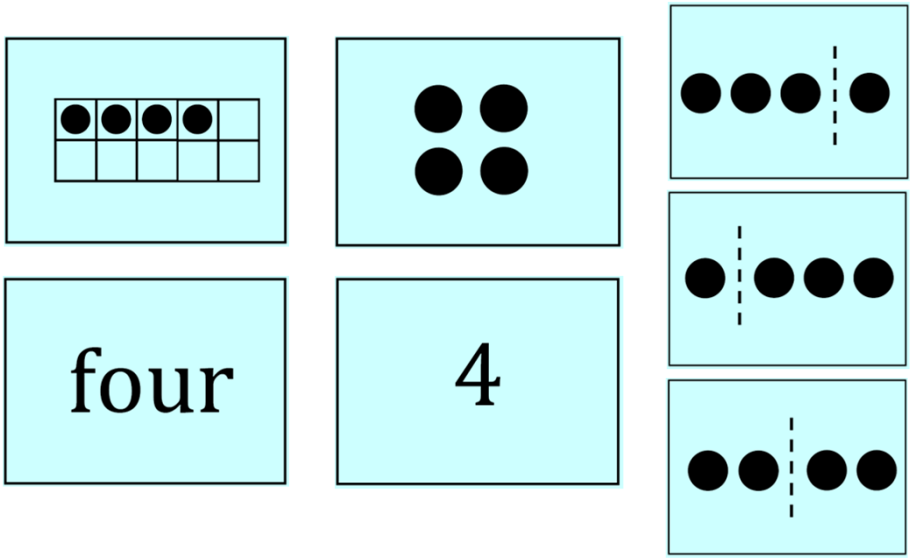
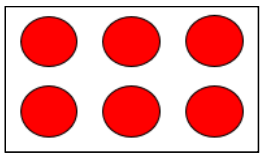
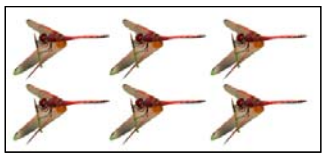
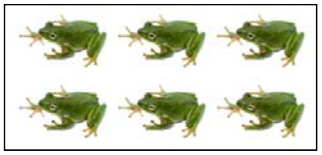
Connecting Representations of Numbers



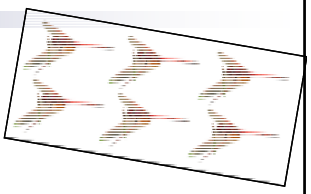
Subitization





Subitization – Tacting a Feature

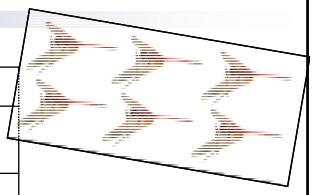


Verbal Conditional Discrimination must be established.

- What is it?
- What part is it?
- How many?

This is complex verbal behavior.

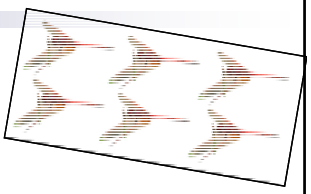
Subitization – Tacting a Feature



Trial	Teacher	Learner
Tact Prompt for Part	Presents item "How many? Six."	"Six"
Tact Transfer	"How many?"	"Six"
Distractor(s)	?	?
Tact Trial Item	Presents item "What are these?"	"Red-veined Dropwing Dragonflies"
Tact Part Check	Presents item "How many?"	"Six"

Error Correction – Run a contrast correction as part of the distract trial sequence

Subitization – Data Collection

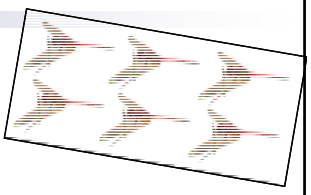


Skills Tracking Sheet

Skill: _____

	Target		Date Introduced	Date Mastered
1	One:	bus		
2		car		
3		red-veined dropwing		
4		plane		
5		frog		
6		circles		
7	Two:	bus		
8		car		
9		red-veined dropwing		
10		plane		
11		frog		
12		circles		
13	Three:	bus		
14		car		
15		red-veined dropwing		
16		plane		
17		frog		
18		circles		
19	Four:	bus		
20		car		

Subitization – Tacting a Feature



Generalization & discrimination should be present for the items in the set.

The concept of quantity has been developed when the individual can subitize (tact) novel items in a set without explicit training.

Cardinality: the size of a set

four reindeer

4 pumpkins

4

four

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Cardinality: the size of a set

This is a setting to ensure the following abilities are generalized across multiple exemplars

- **Words** (saying, writing)
- **Patterns** (identifying, building)
- **Digits** (matching, writing)
- **Assinging units**

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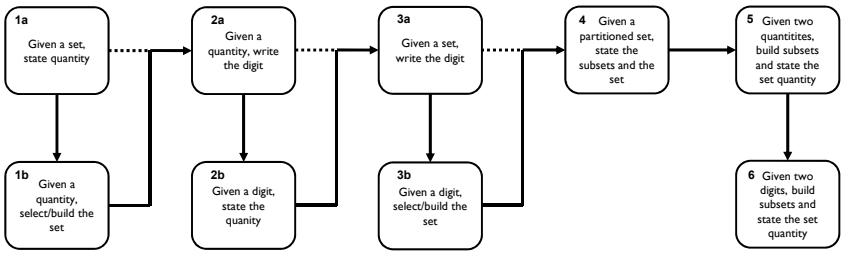
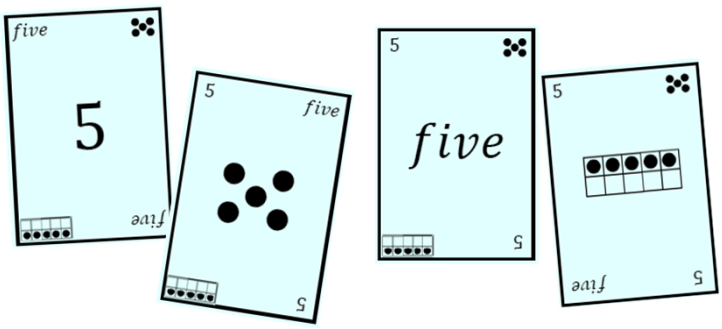
Understanding Individual Quantities



Potential Prerequisites?

What prerequisite skills might students need to learn about cardinality?

Play time!



Match Game

Go Fish!

War!

Other Ideas?

Stable-Order

"consistent count sequence"

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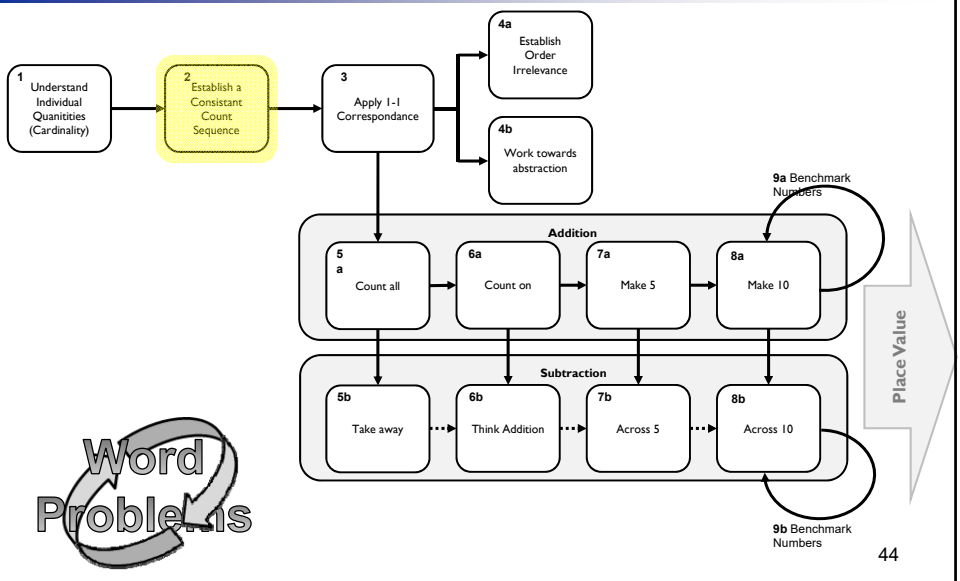
From Quantity to Computation



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Stable-Order: Consisent Count Sequence

- Establish a consistent count sequence.

Once student understand a set of quantities, those are arranged in order of magnitude to establish a count sequence.

Magnitude – Comparing to other sets

Single Comparison - Greater than, less than, equal

Multiple comparisons - Ordering sets

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Single Comparision

- Comparing one number to another

Which is more/less? # or #

2 or **8**

4 or **5**

More bees or trees?

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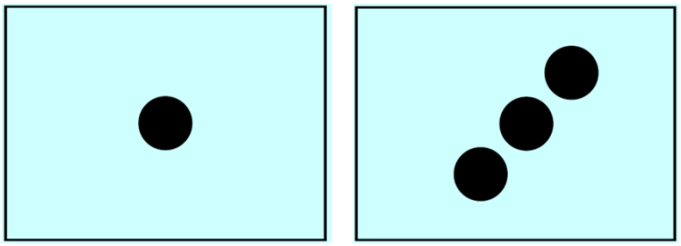
Single Comparision

- Comparing one number to another



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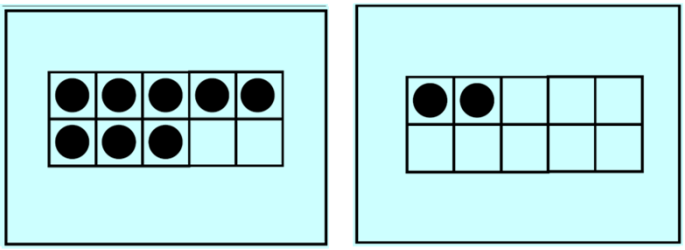
Single Comparision

- Comparing one number to another



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Single Comparision

- Comparing one number to another

greater
less
_____ is _____ than/to _____
equal

The number of bees is _____ than the number of trees.

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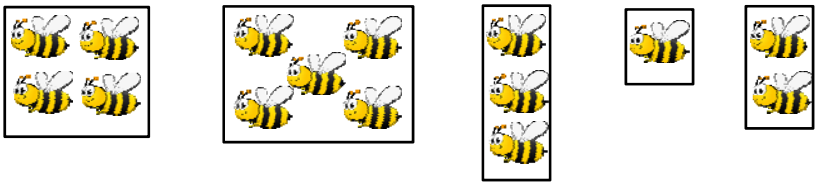
Multiple Comparisions (ordering)

- Comparing more than one number

Least, greatest, minimum, maximum, middle…
ORDERING



1 2 3 4 5

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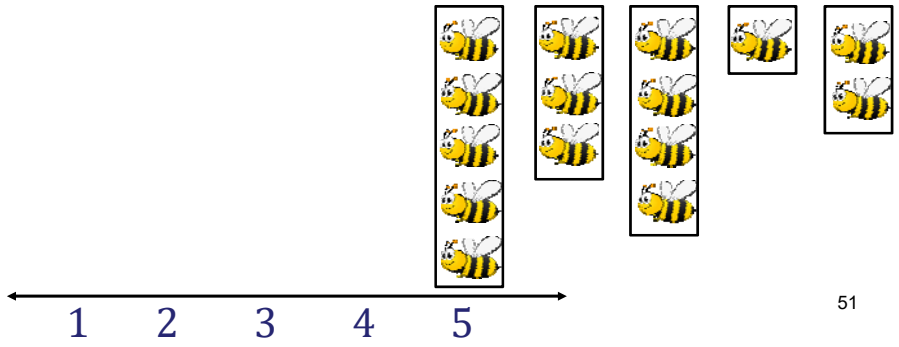
Multiple Comparisions (ordering)

- Comparing more than one number

Least, greatest, minimum, maximum, middle…
ORDERING



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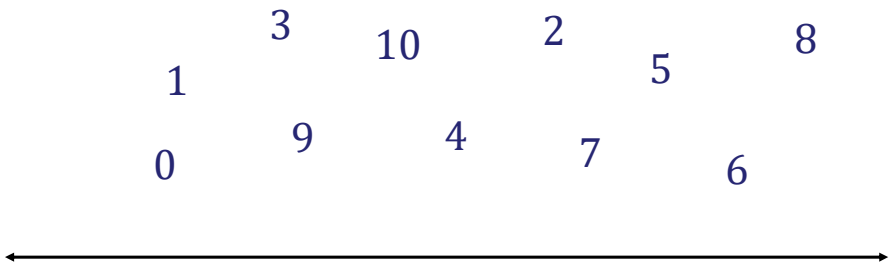
Stable-Order: Consistent Count Sequence



"Zero, *one*, *two*, *three*, *four*, *five*, *six*, …"

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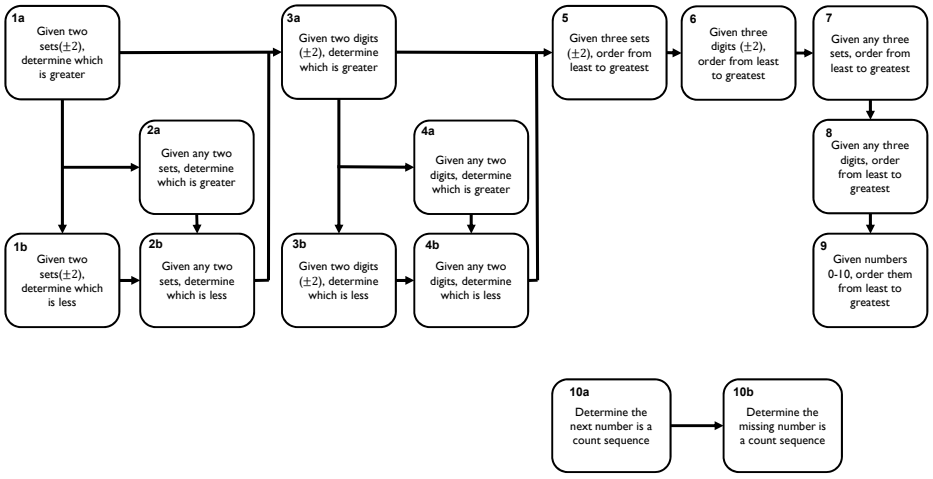
Establist a Consistant Count Sequence



Potential Prerequisites?

What prerequisite skills might students need to establish a consistent count sequence?

1:1 Correspondance

"not just couting words"

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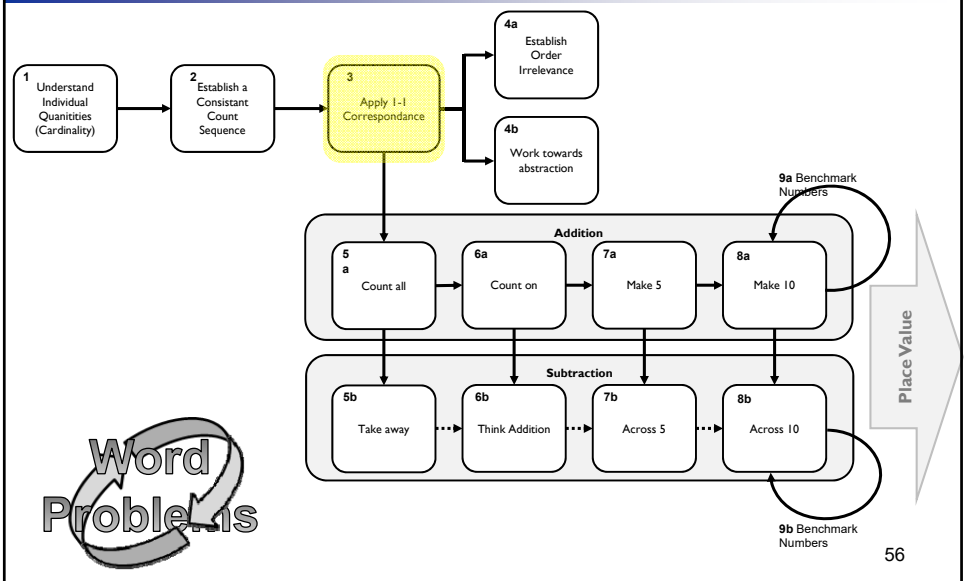
From Quantity to Computation



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1: 1 Correspondence

- Pairing between two sets, each object in A with one and only one object in B

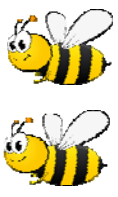
if $\boldsymbol{A = \{1, 2, 3, 4, 5\}}$ and $\mathbf{B = \{a, b, c, d, e\}}$
then $\boldsymbol{A}$ and $\boldsymbol{B}$ are in one-to-one correspondance

$$\boldsymbol{(1, a), (2, b), (3, c), (4, d), (5, e)}$$

1: 1 Correspondence & Cardinality

Cardinality is enhanced through 1:1 Correspondence
Magnitude

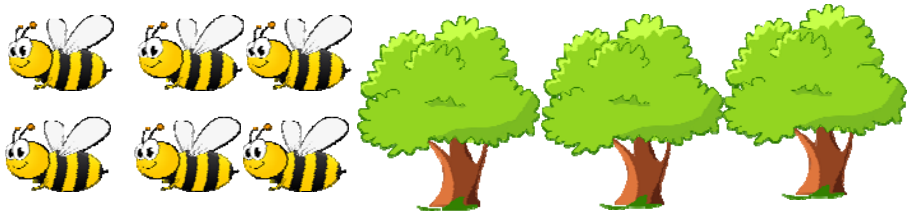
More bees or trees?



1: 1 Correspondence & Cardinality

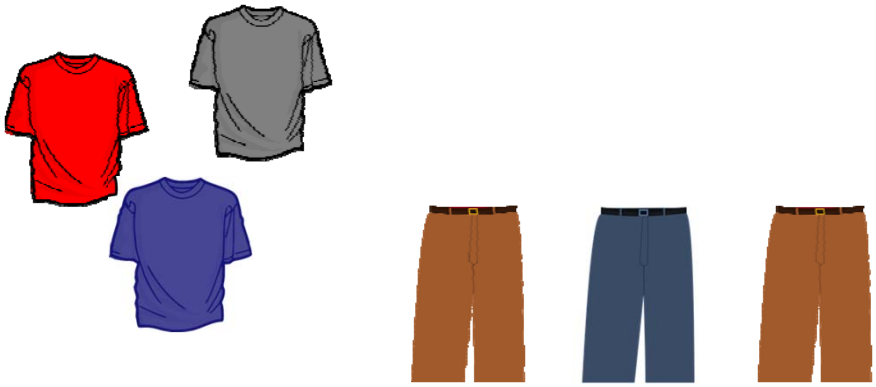
Cardinality is enhance through 1:1 Correspondence…
Magnitude

More bees or trees?



1: 1 Correspondence & Cardinality

Cardinality is enhance through 1:1 Correspondence…
Magnitude



1: 1 Correspondence

- Pairing between two sets, each object in A with one and only one object in B

if $A = \{$  ,  ,  , ... $\}$ and

$B = \{\text{"one"}, \text{"two"}, \text{"three"}, \ldots\}$

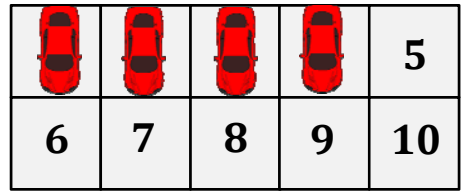
then ...

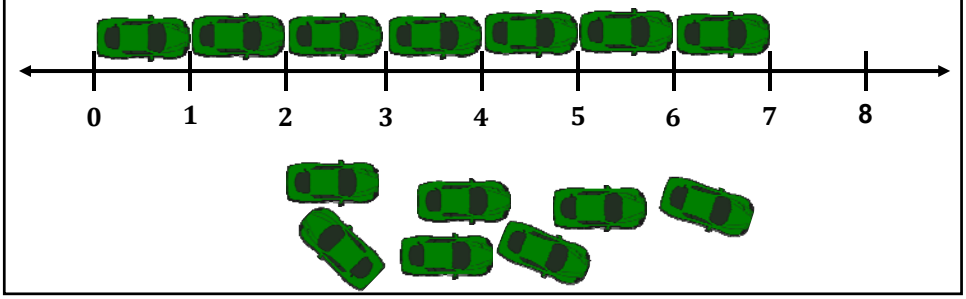
Applying the Count Sequence

- **Partitioning** – Moving from "uncounted pile" to "counted pile" – touching?
- **Tagging** – assigning label; placing in "labeled" spot or attaching label

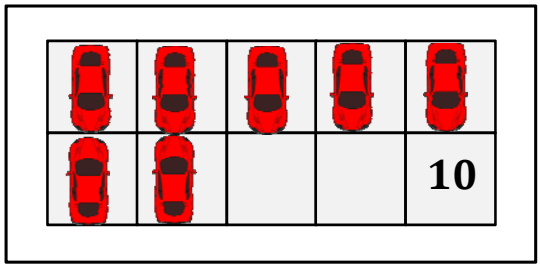
Applying the Count Sequence

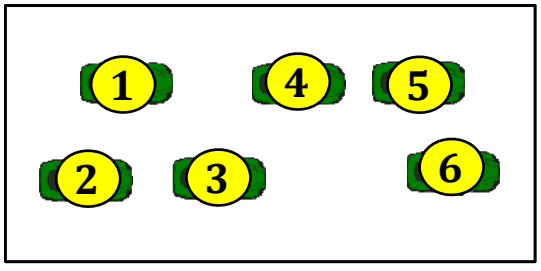
movable





Applying the Count Sequence

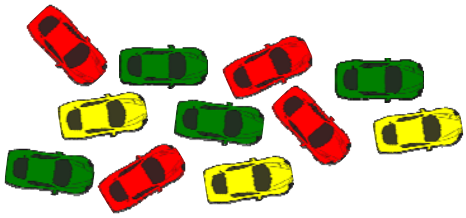




Applying the Count Sequence

movable

1	2	3	4	5
6	7	8	9	10

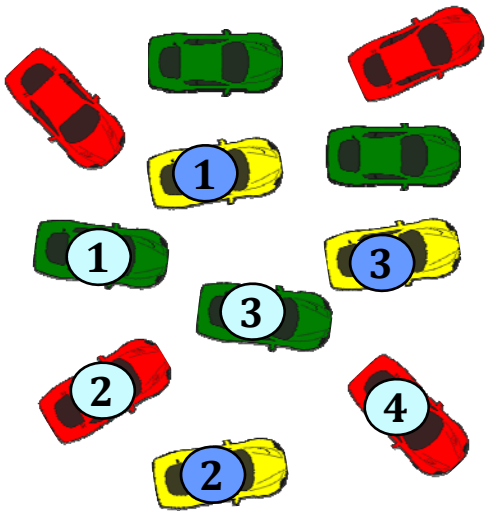


"Get four cars."
Defined set

"How many yellow?"
Discrimination

Applying the Count Sequence

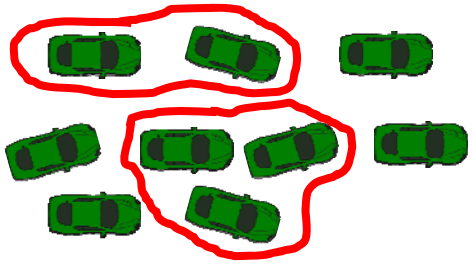
movable



"Get four cars."
Defined set

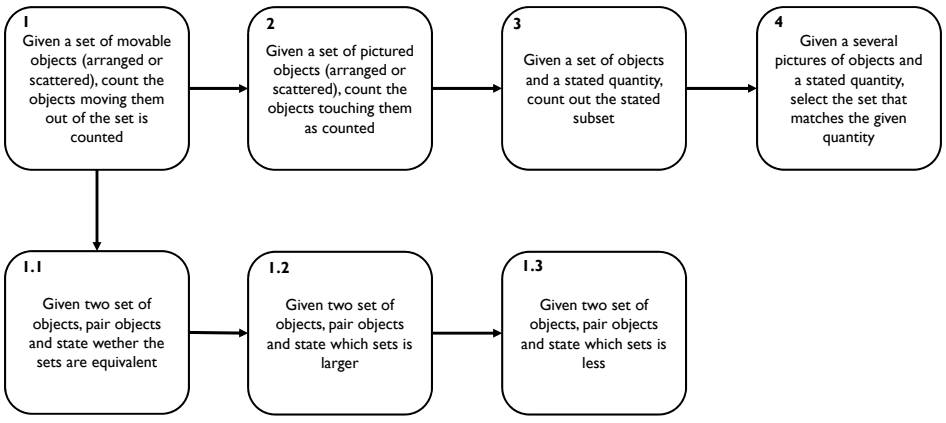
"How many yellow?"
Discrimination

Applying the Count Sequence



$3 + 2 = \_\_\_$

Apply 1: 1 Correspondence



Potential Prerequisites?

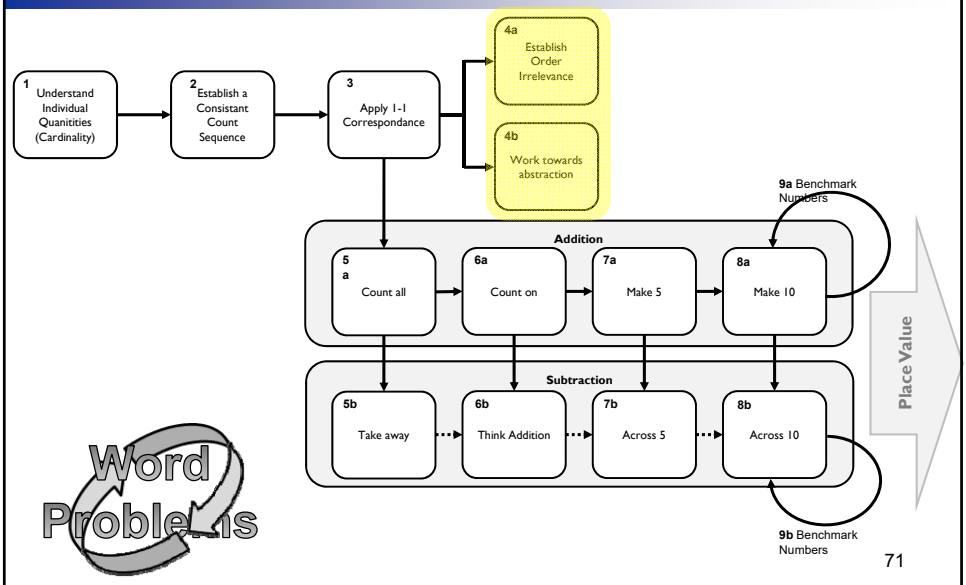
What prerequisite skills might students need to learn about 1: 1 Correspondance?

Order Irrelevance

Abstraction

"what are we counting"

From Quantity to Computation



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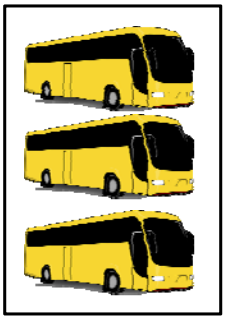
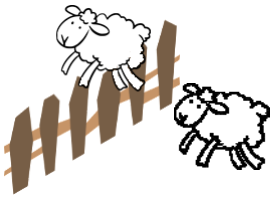
Order Irrelevance

- Counting a set of items in different orders always results in the same size set

Abstraction

- Count things that are "not easily re-counted"





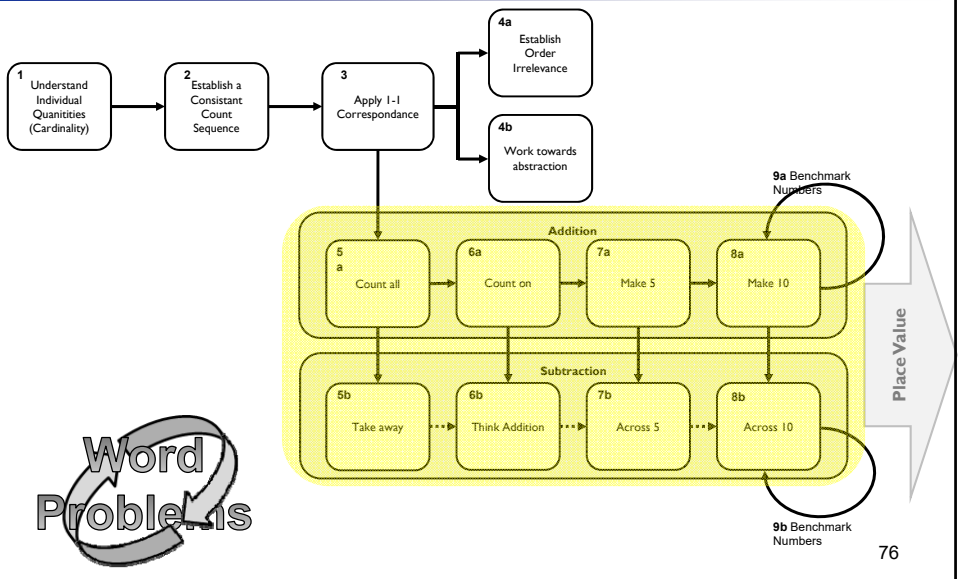
Potential Prerequisites?

What prerequisite skills might students need to learn about flexibly and abstractly?

Operations

Pennsylvania Training and Technical Assistance Network

From Quantity to Computation



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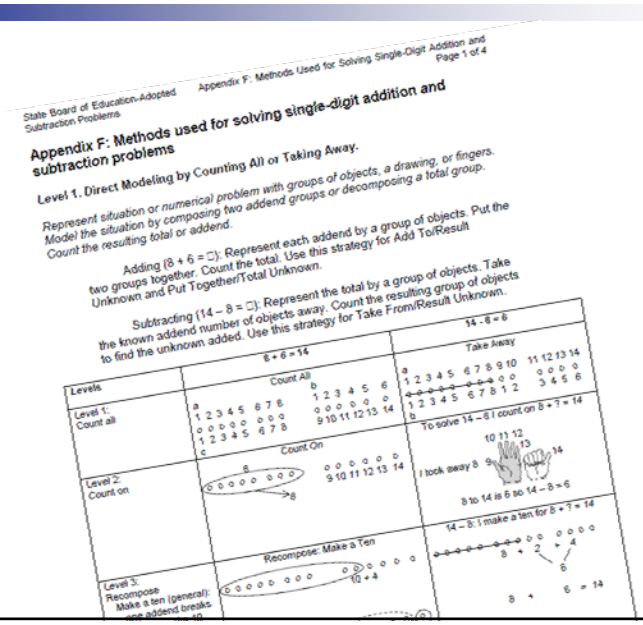
The Mathematics Framework, Appendix F

5 min.



State Board of Education–Adopted Subtraction Problems

Appendix F: Methods Used for Solving Single-Digit Addition and
Page 1 of 4

Appendix F: Methods used for solving single-digit addition and subtraction problems

Level 1. Direct Modeling by Counting All or Taking Away.

Represent situation or numerical problem with groups of objects, a drawing, or fingers.
Model the situation by composing two addend groups or decomposing a total group.
Count the resulting total or addend.

Adding (8 + 6 = □): Represent each addend by a group of objects. Put the two groups together. Count the total. Use this strategy for Add To/Result Unknown and Put Together/Total Unknown.

Subtracting (14 – 8 = □): Represent the total by a group of objects. Take the known addend number of objects away. Count the resulting group of objects to find the unknown added. Use this strategy for Take From/Result Unknown.

Levels	8 + 6 = 14	14 - 8 = 6
Level 1: Count all	Count All	Take Away
Level 2: Count on	Count On	To solve 14 – 8 I count on 8 + ? = 14
Level 3: Recompose Make a ten (general): one addend breaks	Recompose: Make a Ten	14 – 8. I make a ten for 8 + ? = 14

The Mathematics Framework, Appendix F

Levels	8 + 6 = 14	14 - 8 = 6
Level 1: Count all	**Count All**	**Take Away**
Level 2: Count on	**Count On**	**Think +**
Level 3: Recompose Make a ten (general): one addend breaks apart to make 10 with the other addend Make a ten (from 5's within each addend)	**Make 5/10**	**From 5/10**
Doubles = *n*	6 + 8 = 6 + 6 + 2 = 12 + 2 = 14	

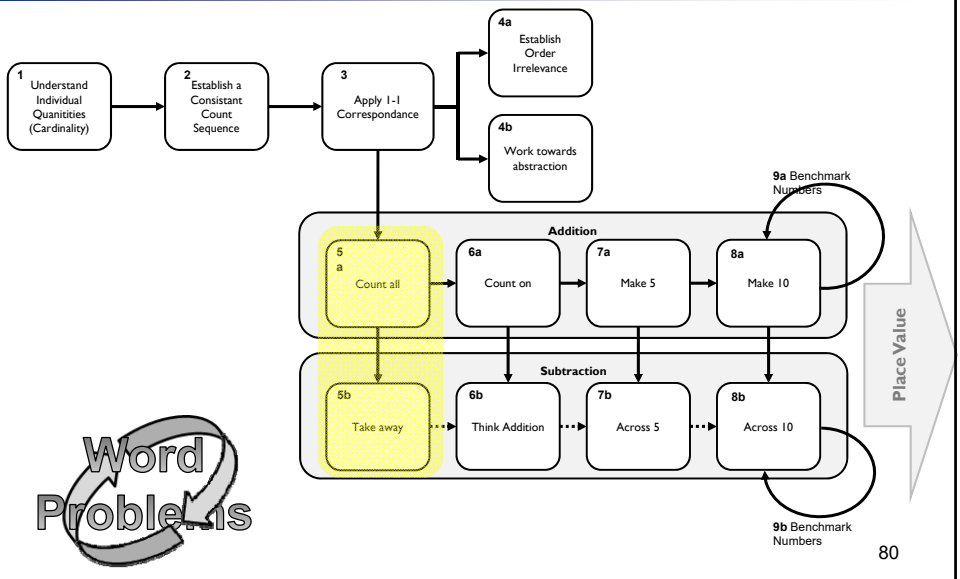
Note: Many children attempt to count down for subtraction, but counting down is difficult and error-prone. Children are much more successful with counting on; it makes subtraction as easy as addition.

Early Addition/Subtraction

"practice counting with symbols"

Pennsylvania Training and Technical Assistance Network

From Quantity to Computation



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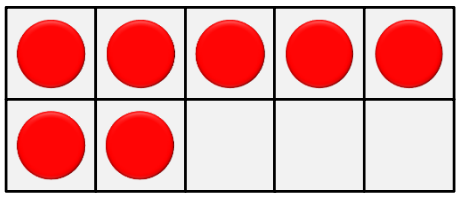
Count All – Take Away

$$4 + 3 = 7$$

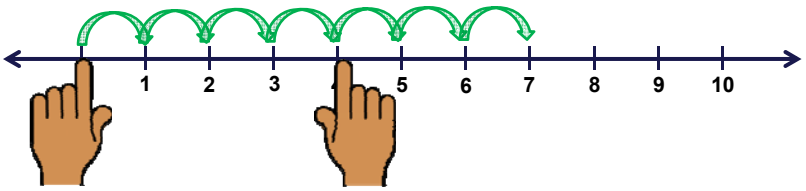
$$(1 + 1 + 1 + 1) + (1 + 1 + 1) =$$



Count All – Take Away

$$4 + 3 = 7$$

$$(1 + 1 + 1 + 1) + (1 + 1 + 1) =$$

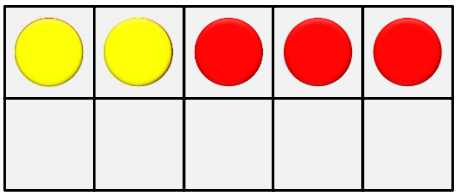


Count All – Take Away

$$5 - 3 = 2$$

$$(1 + 1 + 1 + 1 + 1) - (1 + 1 + 1) =$$

$$(1 + 1 + \cancel{1} + \cancel{1} + \cancel{1}) - \cancel{1} - \cancel{1} - \cancel{1} =$$

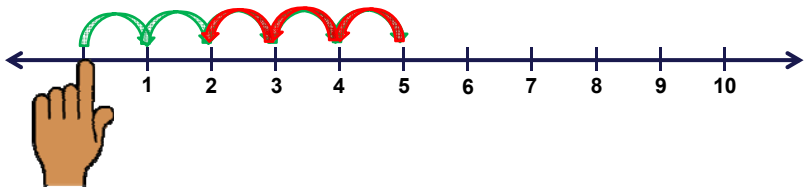


Count All – Take Away

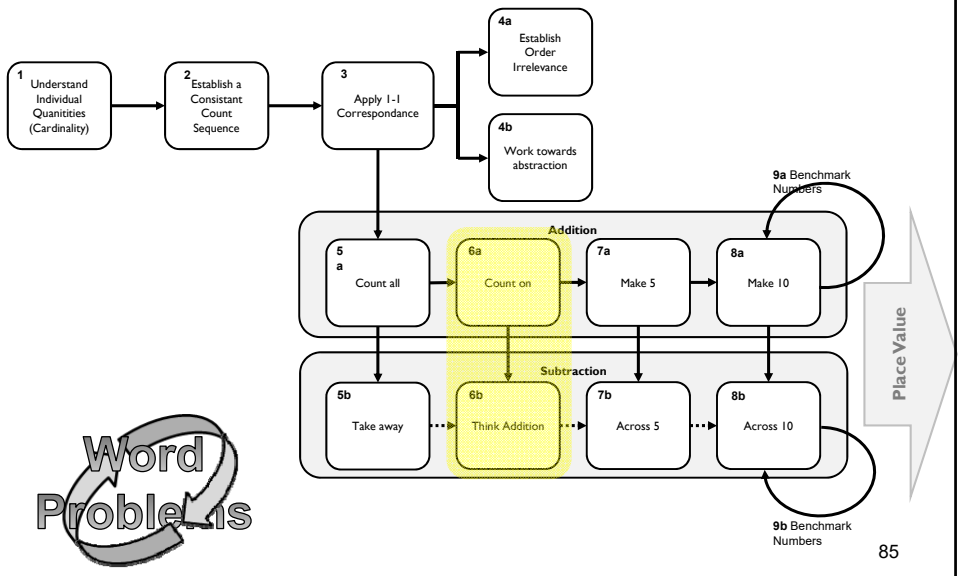
$$5 - 3 = 2$$

$$(1 + 1 + 1 + 1 + 1) - (1 + 1 + 1) =$$

$$(1 + 1 + \cancel{1} + \cancel{1} + \cancel{1}) - \cancel{1} - \cancel{1} - \cancel{1} =$$



From Quantity to Computation



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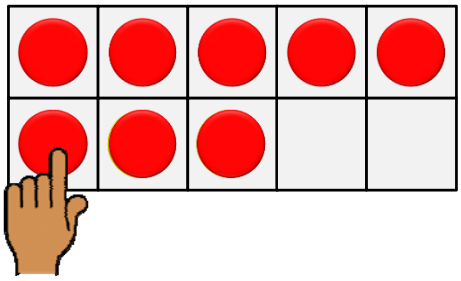
Count On – Think Addition

$$6 + 2 = 8$$

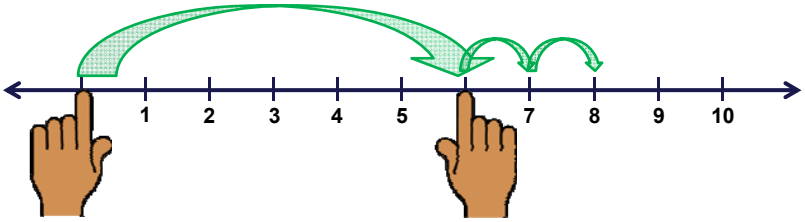
$$6 + (1 + 1) =$$



Count On – Think Addition

$$6 + 2 = 8$$

$$6 + (1 + 1) =$$



Count On – Think Addition

$$6 - 2 = 4$$

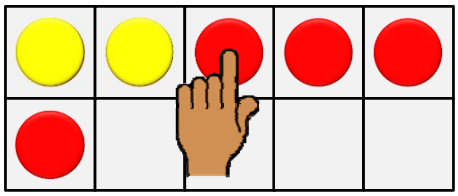
$$6 - (1 + 1) = 6 - 1 - 1 =$$

Count Back

Count On – Think Addition

$$6 - 2 = 4$$

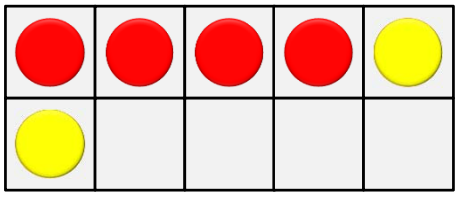
$$2 + \boxed{4} = 6$$



Count On – Think Addition

$$6 - 2 = 4$$

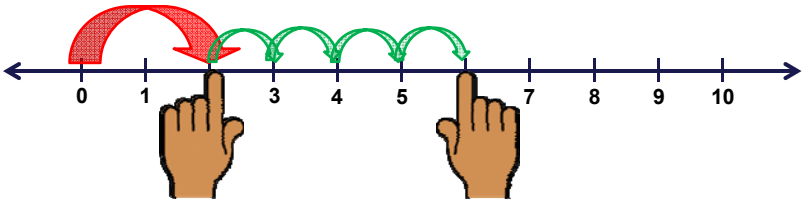
$$2 + \boxed{4} = 6$$



Count On – Think Addition

$$6 - 2 = 4$$

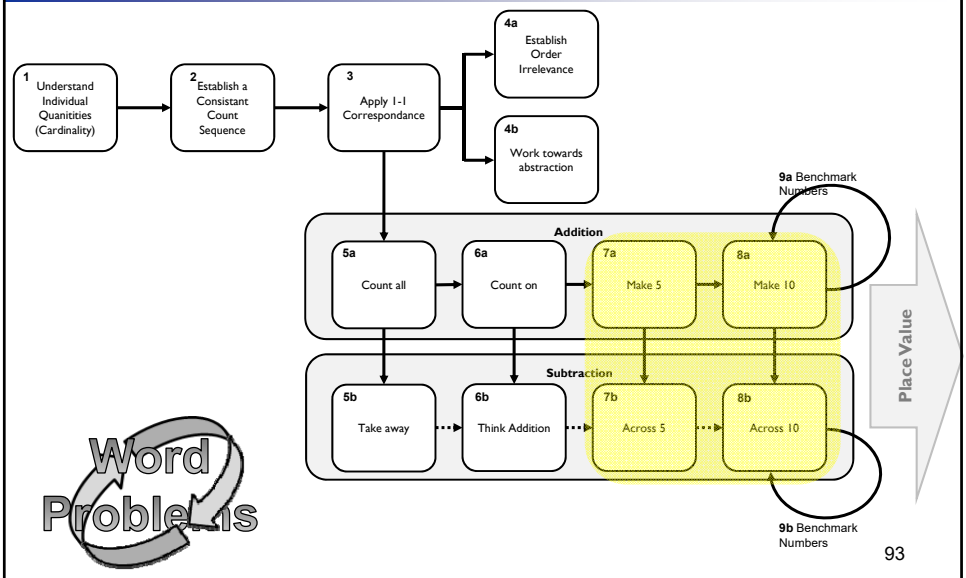
$$2 + \boxed{4} = 6$$



Advanced Addition/Subtraction

"flexibility with numbers"

From Quantity to Computation



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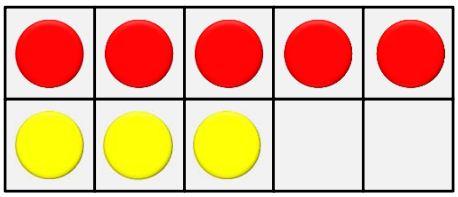
Make 5 – Across 5

$$4 + 4 = 8$$

$$4 + (1 + 3) =$$

$$(4 + 1) + 3 = 5 + 3 =$$

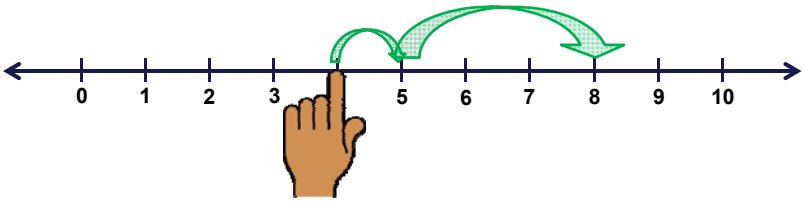


Make 5 – Across 5

$$4 + 4 = 8$$

$$4 + (1 + 3) =$$

$$(4 + 1) + 3 = 5 + 3 =$$

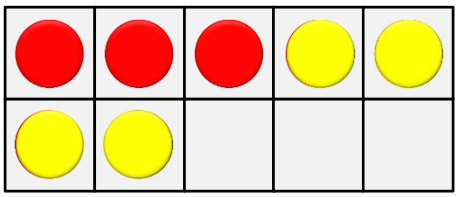


Make 5 – Across 5

$$7 - 4 = 3$$

$$7 - (2 + 2) =$$

$$(7 - 2) - 2 = 5 - 2 =$$

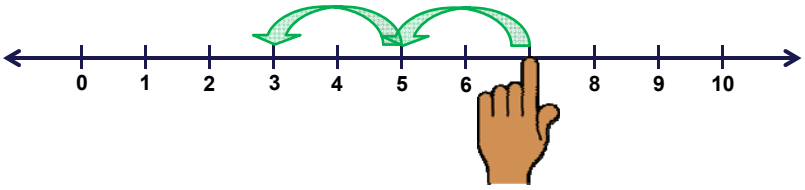


Make 5 – Across 5

$$7 - 4 = 3$$

$$7 - (2 + 2) =$$

$$7 - 2 - 2 = 5 - 2 =$$

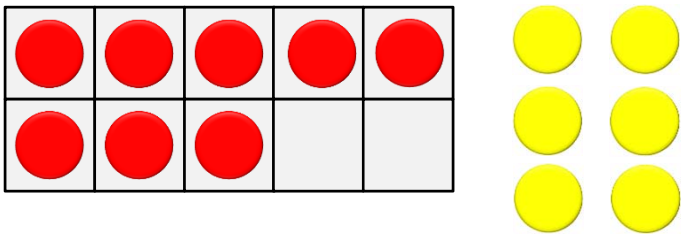


Make 10 – Across 10

$$8 + 6 = 14$$

$$8 + (2 + 4) =$$

$$(8 + 2) + 4 = 10 + 4 =$$

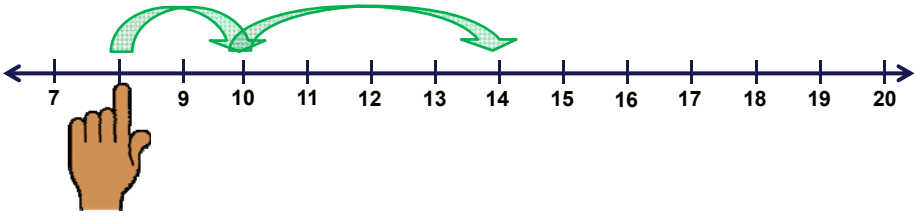


Make 10 – Across 10

$$8 + 6 = 14$$

$$8 + (2 + 4) =$$

$$(8 + 2) + 4 = 10 + 4 =$$

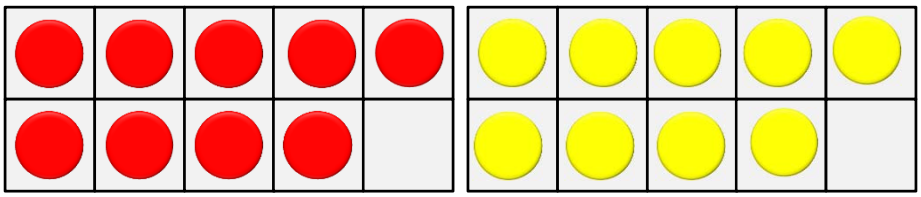


Make 10 – Across 10

$$9 + 9 = 18$$

$$9 + (1 + 8) =$$

$$(9 + 1) + 8 = 10 + 8 =$$

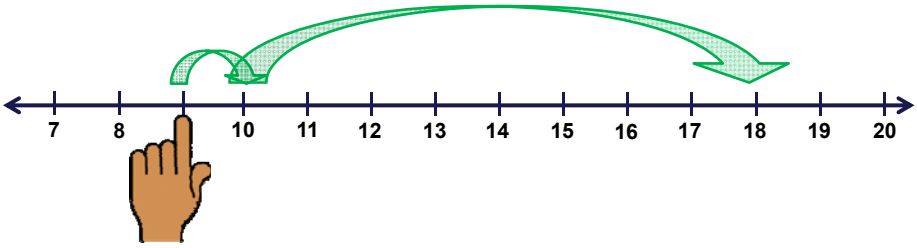


Make 10 – Across 10

$$9 + 9 = 18$$

$$9 + (1 + 8) =$$

$$(9 + 1) + 8 = 10 + 8 =$$

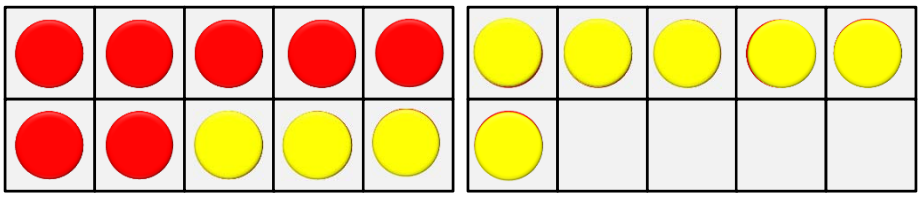


Make 10 – Across 10

$$16 - 9 = 7$$

$$16 - (6 + 3) =$$

$$(16 - 6) - 3 = 10 - 3 =$$

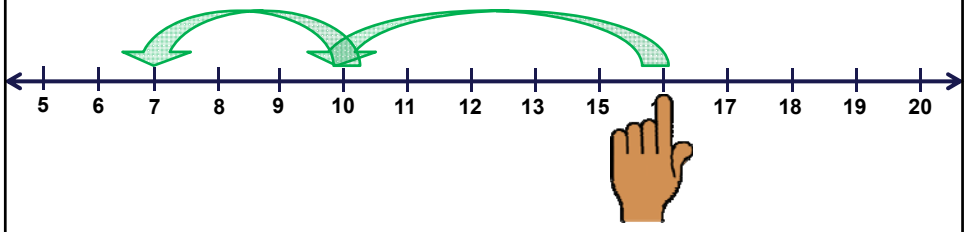


Make 10 – Across 10

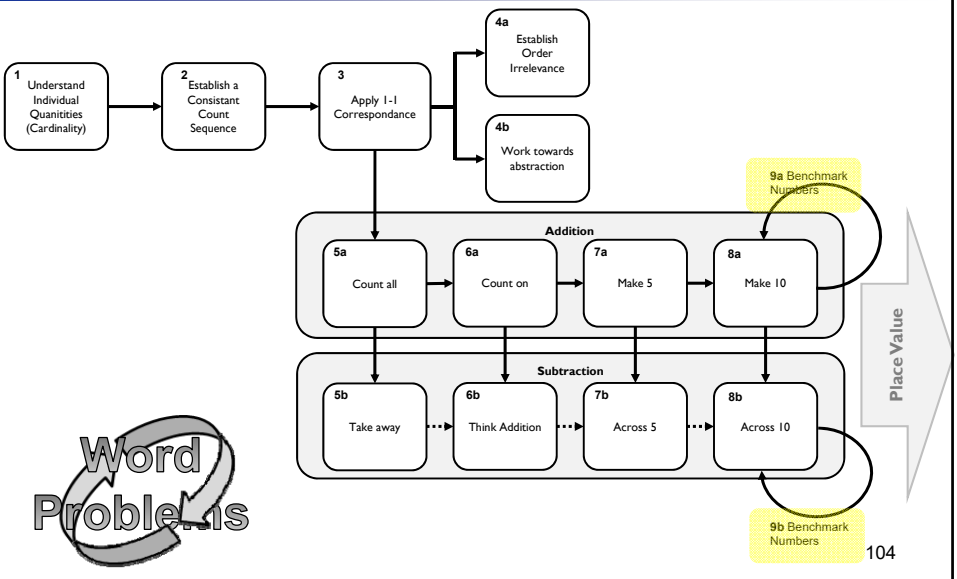
$$16 - 9 = 7$$

$$16 - (6 + 3) =$$

$$16 - 6 - 3 = 10 - 3 =$$



From Quantity to Computation



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Importance of "tens"

- Mental Math is handled in chunks, not an algorithm
- Flexibility with numbers – **NUMBER SENSE**
- Leads to place value

What is **Number Sense?**

"a child's fluidity and flexibility with numbers, the sense of what numbers mean, and an ability to perform mental mathematics and to look at the world and make comparisons"

(Gersten & Chard, 1999)

Importance of "tens"

- Mental Math is handled in chunks, not an algorithm
- Flexibility with numbers – **NUMBER SENSE**
- Leads to place value

$$\begin{array}{r} 34 \\ -\ 16 \\ \hline \end{array}$$

$$(34 - 6) - 10$$
$$(34 - 4 - 2) - 10$$
$$(30 - 2) - 10$$
$$28 - 10$$
$$18$$

Place Value

"seeing sets of 10"

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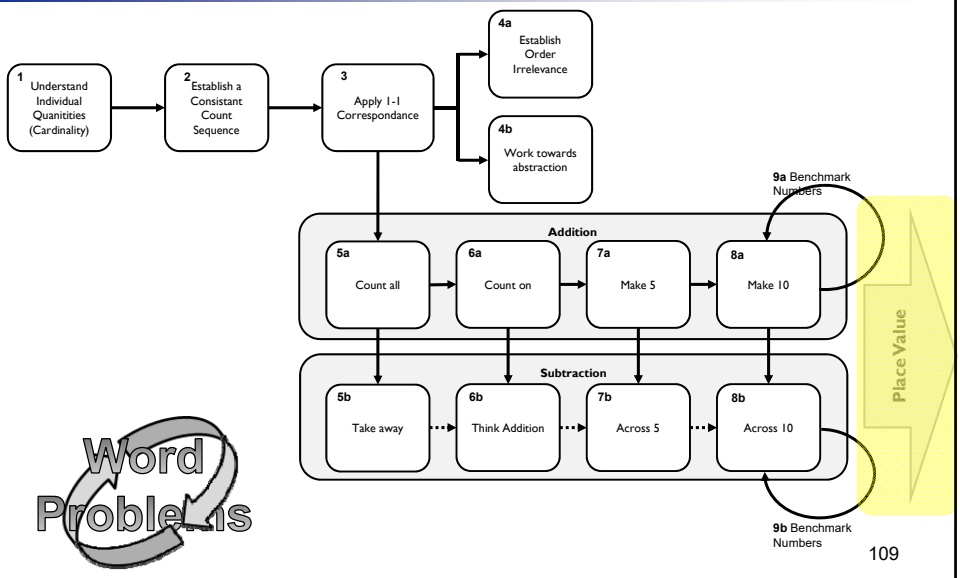
From Quantity to Computation



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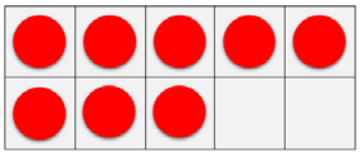
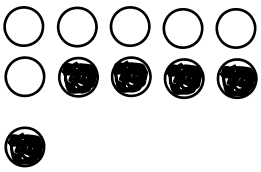
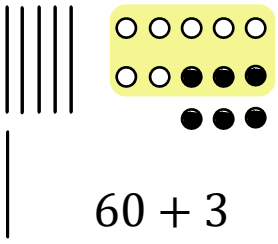
Ten-Frame Progression



$$\begin{array}{r} 1 \\ 27 \\ +\ 36 \\ \hline 63 \end{array}$$

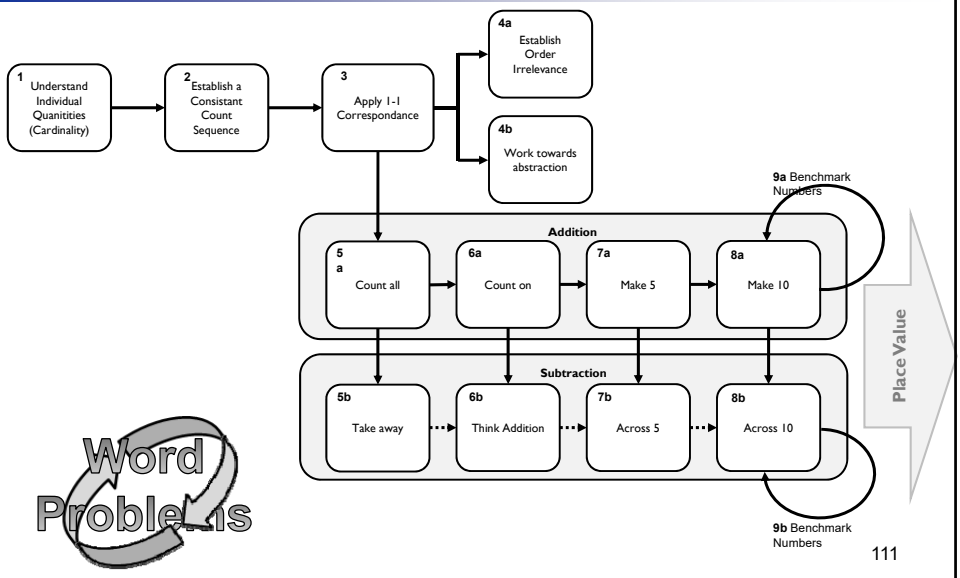


$$\begin{array}{r} 6 \\ +\ 5 \\ \hline 10 \\ +\ 1 \\ \hline 11 \end{array}$$



$60 + 3$

From Quantity to Computation



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Contact Information

www.pattan.net

Educational Consultants

Jared Campbell jcampbell@pattan.net

Willow Hozella whozella@pattan.net

pennsylvania DEPARTMENT OF EDUCATION
Bureau of Special Education
Pennsylvania Training and Technical Assistance Network

PaTTAN

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