The Language of Mathematics: Teaching Quantitative Verbal Concepts

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PaTTAN’s Mission

The mission of the Pennsylvania Training and Technical Assistance Network (PaTTAN) is to support the efforts and initiatives of the Bureau of Special Education, and to build the capacity of local educational agencies to serve students who receive special education services.

PDE’s Commitment to Least Restrictive Environment (LRE)

Our goal for each child is to ensure Individualized Education Program (IEP) teams begin with the general education setting with the use of Supplementary Aids and Services before considering a more restrictive environment.
Quantitative verbal concepts begin with the ability to tact quantities and extends across exemplars. This session will introduce participants to fundamental verbal concepts in mathematics such as quantity, “more”, “less”, and “equal”. Participants will be able utilize a skills sequence to teach these concepts.

Objectives
- Participants will extend their understanding of teaching students to Tact items to build students’ conceptual knowledge
- Participants will be able determine apply quantitative concepts across mathematical domains
- Participants will be able to identify multiple exemplars for mathematical concepts

Session Outline

1. ABA Stuff
2. Concept of Number
3. Two-way Quantitative Verbal Concepts
4. One-way Quantitative Verbal Concepts
Pop Quiz!

Math is a ____________.

Language

Teaching each symbol or Teaching the collection

Each Symbol
• Name – Meaning – Quantity
• Ability to Subitize

Collection
• Counting
• Magnitude
• Applications

Across Exemplars
What is **conceptual understanding**?

**Extended Tacts**

- **Generalization** must occur
  - Can apply to *novel items* without explicit teaching
  - Across…
    1. People
    2. Places
    3. Materials
    4. Instructions
    5. Time

- **Feature/Function/Class**
  - Tacting *critical features* may facilitate concept acquisition

- The tact is involved in the process of *joint control* which assists students in effective *verbal recall* and effective *listener responding*
What is *conceptual understanding*?

**Atomic Repertoires**
- New combination of skills applied to new behaviors
- Most of our spoken language is a result of ARs

**What are the prerequisite skills needed for the atomic repertoires for the math content?**
- Imitation
- Echoic
- Tacts
- Textual Behavior (reading texts/symbols)
- Transcriptive Behavior (copying text/symbols)
- Etc…

*We must identify the skills and outline in a matrix!*  

**Textual Behavior…**

*It is important for students to be able to “read” mathematics.*

However, textual behavior is only relevant when students understand the meaning of the words.

**OR**

Interpreting math symbols is only relevant when they understand their meaning.
## Quantitative Verbal Concepts

“Language used to describe something that is connected to a value (numerical/spatial)”

<table>
<thead>
<tr>
<th>“two-way”</th>
<th>“one-way”</th>
</tr>
</thead>
<tbody>
<tr>
<td>more/less</td>
<td>minimum</td>
</tr>
<tr>
<td>full/empty</td>
<td>greater than</td>
</tr>
<tr>
<td>wide/narrow</td>
<td>volume</td>
</tr>
<tr>
<td>most/least</td>
<td>area</td>
</tr>
<tr>
<td></td>
<td>equality</td>
</tr>
</tbody>
</table>

From this point on...

I am going to simplify the ABA Vocabulary so we can focus on the math.

You can still make connection/improvements if you have that level of background.
Early Quantitative Concepts:

The Concept of Number

“What does three really mean?
What is three-ness”

- MM

Pennsylvania Training and Technical Assistance Network

What does “3” really mean?

3  three  “three”  ○ ○ ○  ○ ○ ○  ○ ○ ○

"1 ... 2 ... 3!"
“one more than 2”  “one less than 4”

“is between...”  “is more than...”  “is less than...”

“is the same as...”

3 units
What is Number Sense?

“a child’s fluidity and flexibility with numbers, the sense of what numbers mean, and an ability to perform mental mathematics and to look at the world and make comparisons”

(Gersten & Chard, 1999)

“Concept Matrix”

<table>
<thead>
<tr>
<th>Teacher (antecedent)</th>
<th>Find digit</th>
<th>Write digit</th>
<th>Write text</th>
<th>Say number</th>
<th>Make pattern</th>
</tr>
</thead>
<tbody>
<tr>
<td>Say number</td>
<td>LR</td>
<td>Trans.</td>
<td>Trans.</td>
<td>Echoic</td>
<td>LR</td>
</tr>
<tr>
<td>Show digit</td>
<td>MtS</td>
<td>Trans.</td>
<td>Trans.</td>
<td>IV</td>
<td>MtS</td>
</tr>
<tr>
<td>Show text</td>
<td>MtS</td>
<td>Trans.</td>
<td>Trans.</td>
<td>Text</td>
<td></td>
</tr>
<tr>
<td>Show pattern</td>
<td>MtS</td>
<td>Trans.</td>
<td>Trans.</td>
<td>Tact</td>
<td>MtS</td>
</tr>
</tbody>
</table>
Teaching each symbol or Teaching the collection

Each Symbol
- Name – Meaning – Quantity
- Ability to Subitize

Collection
- Counting
- Magnitude
- Applications

Subitize

The ability to see a quantity and know how many, without “counting.”

Perceptual and Conceptual
Subitization

Research indicated that dice patterns and rectangular arrays are the easiest for students to learn.

Don’t go crazy!

Subitization – Tacting a Feature

**Verbal Conditional Discrimination** must be established.
- What is it?
- What part is it?
- How many?

This is complex verbal behavior.

<table>
<thead>
<tr>
<th>Trial</th>
<th>Teacher</th>
<th>Learner</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tact Prompt for Part</td>
<td>Presents item “How many? Six.”</td>
<td>“Six”</td>
</tr>
<tr>
<td>Tact Transfer</td>
<td>“How many?”</td>
<td>“Six”</td>
</tr>
<tr>
<td>Distractor(s)</td>
<td>?</td>
<td>?</td>
</tr>
<tr>
<td>Tact Trial Item</td>
<td>Presents item “What are these?”</td>
<td>“Red-veined Dropwing Dragonflies”</td>
</tr>
<tr>
<td>Tact Part Check</td>
<td>Presents item “How many?”</td>
<td>“Six”</td>
</tr>
</tbody>
</table>

**Error Correction** – Run a contrast correction as part of the distract trial sequence
The concept of quantity has been developed when the individual can *subitize* (tact) novel items in a set without explicit training.
What is Number Sense?

“a child’s fluidity and flexibility with numbers, the sense of what numbers mean, and an ability to perform mental mathematics and to look at the world and make comparisons”

(Gersten & Chard, 1999)

Teaching each symbol or Teaching the collection

Each Symbol
- Name – Meaning – Quantity
- Ability to Subitize

Collection
- Counting
- Magnitude
- Applications

Across Exemplars
Stages of Early Arithmetic Learning (SEAL)

Stage 0: Emergent Counting - Cannot count visible items. Either does not know the number words or cannot coordinate the number words with items (one-to-one correspondence).

Stage 1: Perceptual Counting - Can count perceived items but not those in screened collections. This may involve seeing, hearing, or feeling items.

Stage 2: Figurative Counting - Can count the items in a screened collection but counting typically includes what adults might regard as redundant activity. For example, when presented with two screened collections, told how many in each collection and asked how many in all, the child will count from “one” instead of counting on.

Stage 3: Initial Number Sequence - Uses counting-on rather than counting from “one” to solve addition tasks.


Quantitative Verbal Concepts
Quantitative Verbal Concepts (QVC)

“Language used to describe something that is connected to a value (numerical/spatial)”

“two-way”
more/less
full/empty
wide/narrow
most/least
long/short

Lead to...

“one-way”
minimum
greater than
volume
area
equality

Two-way QVCs

“Language used to describe something that is connected to a value (numerical/spatial)”

more/less
full/empty
wide/narrow
most/least
long/short
Two-way QVC – Tacting Adjectives

<table>
<thead>
<tr>
<th>Trial</th>
<th>Teacher</th>
<th>Learner</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tact Trial with echoic prompt</td>
<td>Presents dice patterns of 5 and 2, identical except for attribute.</td>
<td>“More”</td>
</tr>
<tr>
<td>“Let’s talk about more and less.”</td>
<td>“Let’s talk about more and less.” Point to 5. “This one is… more.”</td>
<td></td>
</tr>
<tr>
<td>Tact Transfer</td>
<td>“This one is…”</td>
<td>“More”</td>
</tr>
<tr>
<td>Tact Trial with echoic prompt</td>
<td>Point to 2. “This one is… less.”</td>
<td>“Less”</td>
</tr>
<tr>
<td>Tact Transfer</td>
<td>“This one is…”</td>
<td>“Less”</td>
</tr>
<tr>
<td>Distractor(s)</td>
<td>?</td>
<td>?</td>
</tr>
<tr>
<td>Tact Check</td>
<td>Point to 2. “This one is…”</td>
<td>“Less”</td>
</tr>
<tr>
<td>Distractor(s)</td>
<td>? (include “How many?”)</td>
<td>?</td>
</tr>
<tr>
<td>Tact Check</td>
<td>Point to 5. “This one is…”</td>
<td>“More”</td>
</tr>
<tr>
<td>Continue with tact checks. Vary tact checks with distractors as shown above. End with the below tact check.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Tact Check</td>
<td>“What are these?”</td>
<td>“Planes”</td>
</tr>
</tbody>
</table>

Two-way QVC – Data Collection

<table>
<thead>
<tr>
<th>Target: more/less</th>
<th>Date introduced</th>
<th>Data Mastered</th>
</tr>
</thead>
<tbody>
<tr>
<td>Identical Sets:</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Single Digits</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Dice Patterns</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Tan Frames</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2 Novel Identical</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Planes</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Trucks</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mixed Mastered pairs</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Known with one novel</td>
<td></td>
<td></td>
</tr>
<tr>
<td>NET</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Intraverbal Opposition</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
One-way QVCs

“Language used to describe something that is connected to a value (numerical/spatial)”

minimum
greater than
volume
area
equality

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Commonwealth of Pennsylvania
Tom Wolf, Governor